Entre Mécanique et Architecture

Between Mechanics and Architecture

Edited by
Patricia Radelet-de Grave
Edoardo Benvenuto

avec le soutien de
supported by
La Faculté des Sciences de l'Université catholique de Louvain
La Facoltà di Architettura dell'Università degli Studi di Genova
The Teaching of Stereotomy in Engineering Schools in France in the XVIIth and XIXth Centuries: An Application of Geometry, an "Applied Geometry", or a Construction Technique?

Joël Sakarovitch

Summary: The teaching of stereotomy sends back to the problem of the visualisation in space or the transmission of a tridimensional object by means of a bidimensional reproduction. The analysis of Monge's teaching reproposes the transition from the methods of stoneworkers to descriptive geometry. It parallelly shows the limits of a strictly theoretical teaching in an engineering school such as the École polytechnique.

Résumen: L'enseignement de la stéréotomie nous renvoie au problème de la vision dans l'espace ou de la transmission d'un objet tridimensionnel par une reproduction bidimensionnelle. L'analyse de l'enseignement de Monge fait ressortir le passage des méthodes des tailleurs de pierre à la géométrie descriptive. Il montre également les limites d'un enseignement exclusivement théorique dans une école d'ingénieurs comme l'École polytechnique.

Stereotomy constituted the expert construction technique par excellence, from the Middle Ages until the XVIIIth century. It authorizes variety and complexity of the forms and technical audacities such as hollow-backed staircases or overhanging towers. While giving an appropriate form to each of the stones which compose the vaults, the stoneworkers go on to construct domes, squinches, the undersides of staircases, or an infinite variety of intersecting barrel vaults. Situated at the crossroads between geometry and statics because of its constructive principle, stereotomy becomes, during the Renaissance, the place where the conflicts between architects and master masons are crystallized. The anecdote according to which Philibert de l'Orme if separated from Jean Vaast, master-mason and stoneworker, would have had considerable trouble completing the stairway of the Tuileries, is certainly dubious; but it reveals the tensions, as much as the quarrel which one century later opposed the geometrician and architect, Girard

---

1 UFR DE MATHÉMATIQUE - UNIVERSITÉ RENÉ DESCARTES - ÉCOLE D'ARCHITECTURE PARIS-VILLEMIC, 11, Quai Malaquais - 75272 Paris Cedex 06 (France).
2 These cut stones in the shapes of corners are called "vousoirs".
Desargues, against the stoneworker, Jacques Curabelle. The architect, for the construction of buildings, and the engineer, for fortifications and bridges, are going to be confronted with a problem of the same nature: they cannot ignore stereotomy without becoming dependant on the stoneworkers whom they are supposedly directing. In order to exercise authority on the building site, as well as to be able to imagine projects both innovative and feasible, they must acquire a certain competence in the field of stone and wood cutting.

Thus the education of engineers is going to include an apprenticeship in stereotomy. The richness of this construction technique, the superposition of the problems of a geometric, statics, esthetic, and economic order all met during the vousoir construction, are even going to make, at the time that engineering schools were set up in France, in the second half of the XVIIIth century, one of the key disciplines in their curriculum. But depending on the objective of the training, the type of engineer desired, and the period, extremely dissimilar teachings were included under the same title. Following the evolution of this discipline in the two principal schools of engineering under the Ancien Régime, the Ecole des Ponts et Chaussées and the Ecole du Génie de Mézières, then the Ecole Polytechnique at the time of its creation under the French Revolution and a half century later, is one way of retracing the history of these schools as well as that of construction technique.

**Stereotomy at the Ecole des Ponts et Chaussées**

The instruction given in the first school of French engineers, transformed in 1747 from an administrative department of the Ponts et Chaussées called the “Office of Draughtmen”, maintains, during the Ancien Régime, a sort of intermediary status between on-the-job training which prevailed before, and a scholastic training. Functioning with little groups and founded on the principle of tutors, the Ecole des Ponts et Chaussées included no official professors and offered no lectures. The teaching of stereotomy will only slightly evolve during a half century. The épures become more and more elaborate in the last years of the Ancien Régime. In 1759, the subjects treated by the

---


---

students are still relatively simple (...) a vis saint-gilles (a helicoidal barrel vault carrying a spiral staircase) with splayed circular opening, a sloping barrel vault, or oblique and sloping Saint-Antoine rear vault. The épures questions given at the competition of 1789 were considerably more complex (...) The students accumulated difficulties in order to demonstrate their virtuosity.

But despite this increasing degree of complexity in the subjects of the épures, the teaching of stereotomy remained very near the practice of construction technique. This proximity flows from the general conditions of teaching, but equally from the personality of Perronet, who "reigned" over the Ecole as well as over the corps of engineers. On the other hand as the first director, he was more a man of the arts than a man of science, who allied theoretical prudence and technological curiosity. Let us add that Perronet, to whom we owe some of the most beautiful stone bridges built in France at that period (for example, the Pont de la Concorde and the Pont de Neuilly), is a delicate connaisseur of stereotomy. Used to directing these major constructions, Perronet always insisted on the control of the costs which he considered the responsibility of an engineer as much as the technical constraints. This imperative led him to adjust his requirements in the ways to cut stone as a function of the position of each of the faces of the vousoirs. For large structures, Perronet wanted the stones cut without any paring along the whole length of the bed joints; but in general he didn’t require as good an execution on the heading joints as on the two thirds of their length. Analogously, Perronet accorded more importance to the quality of the mortar used in arched structures. The function of mortar between the vousoirs is in fact double. It naturally increases the adherence between the stones, but it essentially forms a sort of cushion which spreads the enormous pressures which are at stake and keeps the stones from leaning one on the other. Thus the general conditions of teaching in the Ecole des Ponts will have two consequences. The first is to conserve in the teaching of stereotomy a strong inter-penetration of the diverse geometric and practical dimensions, much more important than in the other engineering schools. On the other hand, the teaching of geometry was not the source of any innovation as it was in the case of the Ecole du Génie de Mézières.

---

6 A. PICON, op. cit., p. 135-6.
7 Ibid. p. 54.
8 The bed joints are the faces of the vousoirs perpendicular to the pressures, while the heading joints are, on the contrary, parallel. For example in the case of a vertical wall constructed of cut stone, the bed joints are horizontal and the heading joints, vertical. These remarks of the director of the Ecole des Ponts appear explicitly in the estimates which he established (for example article 75 and 76 of the estimate of the Louis XVI bridge or article 51 of the Pont de Neuilly) or are reported by witnesses and in particular Prony.
Teaching in the Ecole du Génie de Mézières

At the Ecole du Génie de Mézières, teaching of stereotomy was not reduced to the strict utilitarian aspect of the construction technique. The essential objective of this course was the teaching of geometry and the visualization in space. The founders of the Ecole du Génie de Mézières explicitly formulated this idea. Article 9 of the regulations of 1754, probably written by Chastillon, founder of the School, make this point clear: Independantly of the utility of the cutting of stone and wood presented by the different constructions of the King, these arts open such exact and precise knowledge on the drawing of the plans and profiles and on the manner of expressing the relief which must be represented, that one can consider them as Elements (of Euclide). In the foreword of his Treatise on Shadows in Geometric Drawings, Chastillon repeats this idea: We have found nothing more proper for them (the Engineers) than to procure that perfect knowledge of design through the study of stone and wood cutting. Independant of the advantages which result from this study, relative to constructions of which the officers of engineering have the direction, one conceives easily that when one knows how to develop all the faces and knows all the angles of any stone used in a vault, a squinch, etc. (...) or of a piece of timber used in the roof, a dome, a stairway, etc. (...) one has easily the facility to develop a basition, a semi-circle, a cavalier entrenchment, a battery, etc. (...) that when one knows the representation of all these things in order to make them understood by others in the state of the representation as if they were already executed, and to combine the different structures in order to render them as perfect as they possibly can be.

Here the founder of the Ecole du Génie de Mézières expresses perfectly the pedagogic role conferred on stereotomy in this school, the apprenticeship of space that its teaching permits, that sort of intellectual gymnastics susceptible to permit future engineers to mentally represent objects -eventually complex from a geometric point of view- not yet created. This teaching will be, however, judged so fundamental that rather rapidly after the creation of the school, and in any case before the death of Chastillon in 1765, it will be placed in first year, before the course of construction. Here we observe a first sliding in the function of the teaching of stereotomy in relationship to that given at the Ecole des Ponts et Chausées. This sliding is easily explained by taking into account two factors. On the one hand, the Ecole du Génie de Mézières is much more structured, better organized than the Ecole des Ponts et Chausées and a further reflection was led about the nature and the educational value of the teaching given. On the other hand, if one admits that an engineer must receive a certain education concerning the apprenticeship of “visualization in space” then the choice of stereotomy as a vector is certainly judicious, and the arguments advanced by Chastillon are quite pertinent. The case with which the stone workers or the stone cutters are capable of conceiving and tracing the most complex vousoirs is the best proof.

In 1768, when Monge took charge of the teaching of stone and wood cutting, as well as that of geometry, perspective or the drawing of shadows, he only systematized a principle of teaching put in place by his predecessors. Pushing the logic instituted by the founders of the Ecole du Génie de Mézières to the limits, he looked to extract the geometric theory underlying stereotomy and introduce the rudiments of what he would call later descriptive geometry (12) but which he still called the method of cutting stones.

The first stage towards a conceptualization of the graphic techniques seems to be the use by Monge of the methods for cutting stone in other practical or theoretical domains. The first example is given by the deflating problem where he substituted a geometric method for a method using calculation. J. B. Meusnier, without doubt the most brilliant of Monge’s students at the Ecole de Mézières, wrote in 1777 a treatise on the subject, where he explained Monge’s method. He wrote: We have not dwelt upon the details of stereotomy, our readers versed in this part will easily compensate by recalling that they often use the same principles in several épures of stone cutting. Monge used explicitly a similar expression in his treatise On Shadows where he designated by the “rules of stereotomy” the usage of the double projection and of auxiliary plans. Tinseau, who was also one of Monge’s students at Mézières from 1769 to 1771, presented to the Académie des Sciences shortly after he left school, a treatise which contains incidentally the determination of the perpendicular common to two straight lines and of their distance. He points out that he is going to give the construction of the problem by the method used in the cutting of stone and which by its utility, would merit to be better known. And

---

9 CHASTILLON, Projet de règlement sur l'ordre et la police de l'Instruction que le Roy veut et ordonne être donné aux ingénieurs ordinaires admis, volontaires et vétérans dans l'Ecole du Génie établie à Mézières, ma, 17 déc. 1754, archives of corps of Army Engineers, art. 18, sect. 1. §1, carton 1, pièce 9, quoted in B. BELHOSTE, Du dessin d'ingénieur à la géométrie descriptive, l'enseignement de Chastillon à l'Ecole royal du génie de Mézières, in essais, n° 13, Paris, 1990, pp. 102-135; p. 111.

10 CHASTILLON, Traité des ombres dans le dessin géométral, 1764. This undated text was published in T. OLIVIER, Applications de Géométrie descriptive aux ombres, à la perspective, à la gnomonique et aux engrenages, Paris, 1847, p. 5-36. Olivier has estimated that the writing dated from 1775, but Belhoste was able to show that it could be after 1766, that is to say, a year before the arrival of Monge at the Ecole du Génie de Mézières. See B. BELHOSTE, op. cit.


12 J. B. MEUSNIER, Mémorial sur la détermination du plans et jointure de murs en coups de corps d'army Engineers, art. 18, sect. 3, carton 2, 1777.
Tinseau gives a true épure in descriptive geometry, the first which is known for the solution of a geometric problem in space.

Was it Monge who first had the idea of this "transfer of technology", or should one see in the Treatise on Shadows by Chastillon, which was written one year before the arrival of Monge, the first sign of the usage of the process of stone cutting in other technical domains? Certainly Chastillon didn’t use as explicit an expression as did Meusnier and Tinseau. But he gives the convenient and expeditious practices drawn from geometry for drawing shadows and several times quotes as models the courses on stone and wood cutting. That Monge wasn’t the only one, nor necessarily the first one to have the idea to use these processes on other practices, is quite probable. Moreover, Hachette, in the preface to his treatise of 1822, declared that it is up to the masters, to the professors (of the Ecole de Mézières) to whom the honor truly belongs to have led the science of projections to the degree of perfection where it was in 1794\textsuperscript{14}. If he then underlines the special role of Monge, he associates the genesis of descriptive geometry rather largely in the teachings of the Ecole de Mézières and explicitly mentions Chastillon. Be that as it may, the stereotomy courses at Mézières took on a new dimension by becoming the matrix of descriptive geometry.

The teachings of Monge at the Ecole Polytechnique

At the time of the creation of the Ecole Polytechnique, in 1794, Monge had succeeded to some degree in the evolution of a process begun at the Ecole du Génie de Mézières. Taking up again the idea according to which the training of engineers must include an apprenticeship in spatial representation of volumes, and surfaces, and their intersections, he conferred this role, firstly to descriptive geometry and no longer uniquely to stereotomy. A scholastic discipline which was born in a school, by a school and for a school\textsuperscript{15} (but maybe one should say "in the Ecole Polytechnique, by the Ecole Polytechnique, and for the Ecole Polytechnique"\textsuperscript{16}), descriptive geometry allows the passage from one process of training by apprenticeship in little groups which was characteristic of the schools of the Ancien Régime, to an education in amphitheaters, with lectures, and practical exercises, which are no longer addressed to 20 students, but to 400 students. Descriptive geometry also stems from the "revolutionary method". A means to teach space in an accelerated way in relation to the former way of teaching stereotomy, an abstract language, minimal, rapid in the order of stereotomy, descriptive geometry permits a response to the urgent situation as for the education of an élite, which was the case of France at the moment of the creation of the Ecole Polytechnique\textsuperscript{17}.

Thus descriptive geometry occupies at the Ecole Polytechnique, the same place at stereotomy at the Ecole du Génie de Mézières, and one can say at the same time that descriptive geometry is to the Ecole Polytechnique what stereotomy is to the Ecole du Génie de Mézières but also that the Ecole Polytechnique is to the Ecole du Génie de Mézières what descriptive geometry is to stereotomy. But Monge was not satisfied with extricating the essence of the stone workers' drawings. The geometric theory having been disengaged, he presented a theory of stone marking, announced as an example of application of the notion lines of curvature of a surface, where he showed that the method of stone marking of a vault is totally determined by the surface adopted for its intrados\textsuperscript{18}. The vousoirs which constitute the vault must, according to him, satisfy four conditions:

1) The orthogonality of the joints of the vousoir with the surface of the vault. This condition is necessary for static reasons, if not, the action that two consecutive vousoirs exert one on the other, an angle smaller than a right angle would be susceptible to splitting.

2) The orthogonality of the joints of the same vousoir. This requirement responds to the same static problem as the preceding point but also presents an advantage of the esthetic order since thus the lines which divide the vault into courses are perpendicular to those which divide the same courses into vousoirs.

3) The surfaces of the joints of the vousoirs must be developable. The joints also require the most exactitude because (...) it is necessary that the stones touch each other on the largest number of points possible in order that for each point of contact, the pressure is the least and that the approach for all is the most equal (...) it is necessary


\textsuperscript{14} J.-N.-P. HACHETTE, Traité de géométrie descriptive..., Paris, 1822, p. VI.

\textsuperscript{15} For the notion of scholastic discipline see for example A. CHEVERE, L'histoire des disciplines scolaires: réflexions sur un domaine de recherches, Histoire de l'éducation, n°38, INRP, mai 1988, pp. 59-119.

\textsuperscript{16} Monge taught at the Ecole Normale from the year III and at the same time at the Ecole Polytechnique, but the sudden closure of the Ecole Normale, the impossibility to teach analytic geometry there, made the Ecole polytechnique where he was the master, the place where he fully expressed his conceptions concerning descriptive geometry.

\textsuperscript{17} On this subject see also N. et J. DHOMBRES, Naissance d'un pouvoir: sciences et savants en France, 1793-1824, Paris, 1989, pp. 417-421.

The Teaching of Stereotomy in Engineering Schools in France

Ponts, he seemed designated to teach stereotomy. But Prony entrusted Eisenmann with the teaching of mechanics and gave stereotomy to Bruyere who had only been slightly influenced by Monge.  

The teaching of Stereotomy at the Ecole Polytechnique, during the second half of the XIXth century

The limits of the theory of the stone marking of vaults according to the lines of curvature will appear clearly in the years 1820-1830 with the construction, in England first, then in the rest of Europe, of a network of railways and the construction of oblique bridges. Submitted to the problems of weight and of shaking produced by the passage of trains, these bridges, whose platforms were not perpendicular to that of the crossed way, posed some very delicate stone marking problems as the obliqueness increased. 

In the article which marked the departure of the theoretical studies on the subject, Lefort quoted entirely the passage from Monge’s lecture concerning stone marking of vaults to confirm that this well made analysis, rendered the mechanical element which dominates the question completely abstract. It is necessary, in fact, (...) to direct the surfaces of the joints normally to the surface of maximum pressure. But this surface, for a vault whose extrados is parallel to its intrados is the normal surface of the line of the biggest contraction, a line which does not mix necessarily with the line of the smallest curve of the surface. The different solutions proposed, and in particular the helicoidal one adopted in England (fig. 2), determine course and joint lines (which) differ notably from the lines of curvature which the exclusive consideration of the geometric part of the question accepted by Monge. We shall notice, however, that the condition itself of having right angle joints in the vault is more geometric than mechanical, and the true definition of the joint surface would be the following: that surface must be such that the normal at a given point should be directed according to the resulting pressures on that

---

22 The obliqueness is even bigger as the angle between the two tracks is small. As long as the angle is not inferior to about 65°, once can do the stone marking without major inconvenience, the bridge being a straight cylindrical vault: the big obliquity corresponds to the angles inferior to 40°. The first stone bridge including such an angle (39°) was built by Chapmann, for the Grand Canal of Ireland (near Nass) around 1787. The record seems still to be held by the bridge built in 1830 on the river Gaulestes, for the Stockton-Darlington railway with an angle of 27°.
point\(^{24}\). Of course Monge could not have had in mind a problem which was posed, as we have already said, posterior to his course. Nevertheless, a restriction clearly appears here to a generalization of the solution proposed by Monge which is only entirely satisfying when the surface of the vault is such that the line of the minimum curvature passing by each of the joints (can) be drawn completely on the intrados in the upper part of the abutments.

Monge’s theory gives a purely geometric solution to a problem which is also -maybe above all- static and economic. The practical result will be that the “optimal” solution to the problem posed will be found in England, a country where, it is true, the questions of oblique bridges was first posed. That England was ahead of France in the beginning of the XIXth century in the metallurgical industry is one thing. But that on the specific problem of stereotomy- a field in which France had benefited from a long tradition of corporations, of incommensurable know-how over any other European country, with an advance of two centuries in relation to England as far as the edition of the first treatise\(^{25}\), and where geometric theory was first found explicitly mentioned subjacent to the stoneworkers’ drawings - that the solution was British, was felt as a failure among the corps of engineers. A strong resentment was expressed in numerous articles against Monge\(^{26}\).

This example not only brings into question the theory of stone marking professed by Monge, but more generally, Monge’s vision of stereotomy. Descriptive geometry theorizes stereotomy quite well, but in a double sense: it expresses the subjacent theory of the treatises and drawings of the stoneworkers, but it also makes stereotomy theoretical. It is no longer “practical” stereotomy which is taught at the Ecole Polytechnique during the first half of the XIXth century, but an intermediary discipline, disconnected from the real problems met in vousoir constructions and whose concrete character only appears in relation to descriptive geometry.

By presenting a problem from a purely geometric angle, whose complexity comes from the multiple fields of knowledge which interfere, the teaching of descriptive geometry will give engineers a false vision of the profound nature of the solutions to be used and a scorn of the knowledge and the drawings proposed previously by the stoneworkers. Far from enlarging the range of possible constructions in vousoir architecture by the renewal of the realizable surfaces, descriptive geometry will rather contribute to the decline of technique, and in other respects a loss in speed for multiple reasons. By placing stereotomy in the framework of construction courses, by speaking first of materials and the static problems, the English avoid putting stereotomy under the steam-roller of geometry. But after Monge’s course, it is finally stereotomy without mortar, without cost, and even, in a certain sense, without pressure which is taught.

We must wait until the 1850’s and the arrival at the Ecole Polytechnique of La Gournerie in the chair of descriptive geometry and its diverse applications including stereotomy, for this teaching to be profoundly modified. The principal adversaries to Monge’s theory of stone marking came from the corps of the Ponts et Chaussées, of whom La Gournerie will be one of the most violent detractors. The first important modification, introduced by La Gournerie in 1852, consisted precisely in separating stereotomy from descriptive geometry and putting it in the second year program. The autonomy of teaching of stereotomy naturally permitted it to be attached equally well to the course on construction, and to evoke the practical problems of construction which had been totally thrown out.

La Gournerie also gave an important place in the stereotomy course to oblique vaults and killed two birds with one stone; first by demonstrating the dangers of Monge’s “over-geometrization” of practical problems. It is naturally this point which could explain the importance given to the subject of oblique bridges at the Ecole Polytechnique, a subject which, we must recognize, should rather find its place in the school of application concerned. Second, he introduced in the stereotomy course the only problem which was really current, the subject of numerous articles, sometimes divergent, in France as well as abroad (essentially in England)\(^{27}\). If the engineers of the Ponts et Chaussées who studied the problem of oblique arches in the XIXth century agreed among themselves to condemn on this occasion, Monge’s theory, they kept divergent opinions about the direction of the pressures in such an arch or about the fact of knowing if this direction was a function or not of the stone marking adopted. Moreover, during the Universal Exposition of 1878 in Paris, La Gournerie presented an ingenious experimental device capable of answering all these questions for which he was awarded a gold medal (fig. 3).

With La Gournerie, the teaching of stereotomy at the Ecole Polytechnique found a certain equilibrium. At the same time taking advantage of the descriptive geometry course from the purely geometric point of view, he also integrated the technical and practical constraints into a course of construction. But ironically, this equilibrium was attained at


\(^{25}\) The first treatise on stone cutting was written by Phillibert de l’Orme, edited in 1567; the first work in the English language on this subject by General Vallencey, was only published in 1666.

\(^{26}\) The article by P. Lefort, (Lefort, 1839), is the first where a systematic criticism of the theory of stone marking using the lines of curvature appeared. The point of view of Lefort is taken up again by La Gournerie, and by Athénar, Graeff, etc.

\(^{27}\) In his Dissertation on the stone marking of oblique arches, published in 1870, La Gournerie made a synthesis, listing over 50 references on the subject.
the moment when the technique of construction became obsolete, with the appearance of concrete and the development of metal architecture.

Fig. 1 - Ellipsoidal vault using lines of curvature

Fig. 2 - Oblique bridge helicoidal or "English" solution
Fig. 3 - La Gournie's experimental device