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A Cross-Disciplinary Comparison

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CHAPTER THIRTEEN

FROM ONE CURVE TO ANOTHER OR THE PROBLEM OF CHANGING COORDINATES IN STEREOTOMIC LAYOUTS

Joël Sakarovitch

The "sloping vault" is not *a priori* the most difficult mortarless piece of the stereotomic repertoire. Squinches, domes, spiral staircases in the style of the *vis de Saint-Gilles*, or vault penetrations are no doubt more difficult pieces to carve and, as is often the case, to draw.

However, the sloping vault poses greater representation problems, due to the choice of reference system, and allows one to grasp the methods authors use to approach a mortarless piece of architecture, if indeed they use a method. The sloping vault offers numerous solutions and, as we shall see later, many opportunities for making mistakes, which are

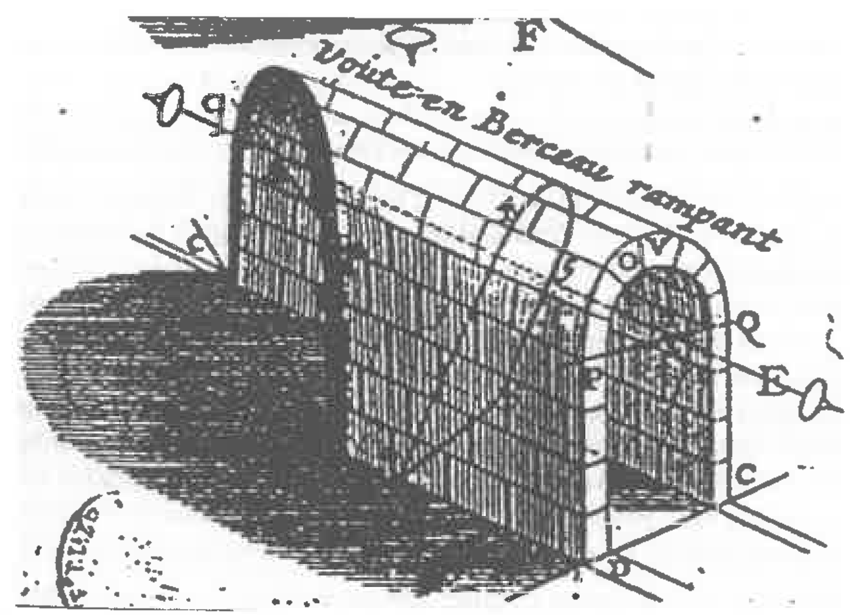

Fig. 13.1: A sloping vault, drawing from A. Bosse in [1], pl. 2.

always more revealing about the methods employed than correct drawings. It must be added that the choice of "sloping vaults" is not mine to begin with but Girard Desargues'. A 17th century mathematician and architect, this author is indeed partly characterized by having written a small opuscule entirely devoted to the study of this particular vault. If Desargues claims to present a "universal method" and only studies one vault, it is because, as I shall try to demonstrate, the corresponding blueprint indeed offers great geometrical wealth.

The authors

Before giving a detailed description of the mortarless architectural piece that will serve as a guide in this study, I would like to justify briefly why I chose the authors I wish to study, besides Desargues. The treatises of Philibert de l'Orme, Jousse, Derand and Frézier are, together with that of Jean-Baptiste de La Rue, the major French treatises on stereotomy. Hence it is not the presence of the first four that must be justified but rather the absence of the latter, and, to a lesser extent, that of Bosse or Millet de Challes. The reason for this is simple: the five authors chosen all study sloping vaults and present radically different solutions whereas the other authors only reuse one of their predecessors' solutions, at least in this particular instance.

Philibert de l'Orme

Philibert de l'Orme (1514–1570) is one of the most famous French architects, for both his built and written opus, as well as his influence on the history of architecture. He did not publish, as did his successors, a treatise on stone carving but a treatise on architecture [cf. 10], in which Books III and IV are devoted to stereotomy, a construction technique that is given its rightful place in his global theory on architecture. He thus inaugurated a new conception of treatises since, as he put it, "geometrical drawing [has] been used in architecture neither by the men of antiquity nor those of modern times" [10, Fol. 87].

Desargues

Although he was also an engineer and an architect, Desargues (1591–1661) has above all remained famous for his mathematical opus. His main contribution, the *Brouillon proiect d'atteinte aux événements des rencontres du Cône avec un Plan,* published in 1639, makes Desargues the father (or to be precise, the grand-father) of projective geometry. It is therefore to one of the greatest geometers of his time that we owe the small opuscule entitled the *Brouillon proiect d'exemples d'une manière universelle du S.G.D.L. touchant la pratique du trait à preuves pour la coupe des pierres en l'Architecture,* comprising four pages of text and four pages of figures.[1] Desargues did not write a treatise on stone carving and, as I said earlier, only studied sloping vaults in these few pages, by taking the most general point of view possible from the start.

Jousse

While Philibert de l'Orme or Desargues have, for various reasons, remained famous, Mathurin Jousse (1607–1650), master mason of La Flèche, is today quite unknown. The only construction attributed to him, the organ gallery of the Jesuit church in La Flèche, a true masterpiece of stereotomy, is now thought to be perhaps the work of another. A part from *Le secret d'architecture découvrant fidèlement les traits géométriques, coupes et dérobements nécessaires dans les bâtiments*..., published in La Flèche in 1642, we owe him *Le Théâtre de l'art du charpentier* (1650) and *La Fidèle ouverture de l'art du serrurier* (1627). Gabriel-Philippe de la Hire had the last two works reprinted in 1702, which goes to show the importance he gave them. While Philibert de l'Orme wrote for architects and Desargues for "excellent contemplative individuals", Jousse clearly wrote for master masons, stone carvers and "any builders who do not possess the broadest experience in this Science (namely Geometry)" [6, Introduction].

Derand

In 1643, François Derand (1588–1644) published *L'Architecture des voûtes ou l'Art des traits et coupe des voûtes*..., a treatise that enjoyed great success. Having entered the Jesuit order in 1611, Derand was both a mathematical teacher and an architect for his order. In Paris, he designed the Saint Paul-Saint Louis church and the Jesuit convent that has nowadays become the Lycée Charlemagne. The success of his architectural work is well deserved as it makes genuine progress

[1] A fifth figure page is part of the opuscule but pertains to the treatise on perspective published in 1636.

compared with his predecessors. Indeed, it displays almost all the different architectural devices used in the 17th Century. The blueprints are very carefully explained and far easier to read than those of the preceding treatises.

The 1640's were particularly fruitful for French treatises on stereotomy since the publications of Desargues, Jousse, Derand and Bosse are quasi contemporary. Such simultaneity is of course not fortuitous, the 17th Century being the golden age of mortarless architecture in France. One might mention for example the Louvre by Lemercier, the Sorbonne chapel (also by Lemercier), the Val de Grâce church (by Mansart and then Lemercier), the Paris Observatory built by Perrault or this masterpiece of French stereotomy, the hall of the Arles Town Hall, by Hardouin-Mansart. Among all the architectural works studied and listed by Pérouse de Montclos, half were built during the 17th Century [cf. 13].

Frézier

Amédée-François Frézier (1682–1773), a military engineer in charge of studying the means of defending the West Coast of Latin America against English attacks during the War of Spanish Succession, was nominated director of the Brittany fortifications in 1739. We owe him the great French stereotomy treatise that predates Monge's teaching course, *La Théorie et la pratique de la coupe des pierres et des bois pour la construction des voûtes... ou traité de stéréotomie à l'usage de l'architecture.* Published between 1737 and 1739, this work is very different from earlier treatises. The first tome, an initiation to three-dimensional geometry, opens on a Vitruve quotation boasting the merits of geometry, and starts with an ardent apology of the "usefulness of theory in the arts relating to architecture". Devoted to a theoretical and abstract study of spheres, cones, cylinders and their intersections, it is presented as a succession of theorems, corollaries and lemmas. But in spite of the theoretical nature of this study, Frézier takes great care to convince his readers that none of the problems studied should escape the attention of anyone interested in stereotomy. Each problem is immediately followed by a "practical application" that justifies the study.

As a mirror to his "practical applications", which follow general theorems in the first tome, the drawings proposed in the two other tomes are followed by "explicative demonstrations" that attempt to justify the graphical constructs with geometry. Although these attempts

at demonstration are often not very convincing and quite incomplete, the approach is essentially novel. Frézier did not propose a geometry theory but inaugurated what Gino Loria called "scientific stereotomy" [cf. 9]. Indeed, after this publication, he began to be recognized as an authority on construction and stereotomy. René Taton [cf. 16] is nonetheless right in emphasizing that the wealth of the text is more easily perceived by readers with a knowledge of descriptive geometry than by the author's contemporaries for it is difficult to distinguish new methods in this imposing treatise of 1500 pages, drowned as they are in a sea of processes of very unequal interest.

Sloping vaults

The point of the blueprints studied here is to provide the layout necessary for the construction of a vault against a wall. In the simplest—and most common—situation, the wall is vertical, and the axis of the vault horizontal and perpendicular to the wall. In this case, elevation is sufficient and no blueprint is necessary to build the vault. The situation of interest here is, on the contrary, the most general possible. I know of no such example in architecture and it can be considered as a case study. However, one of the oculi of the Seville cathedral provides an example of a mortarless cylinder with any axis direction relative to the adjacent wall. The givens are the positions of the wall and the vault cylinder, their situation with respect to one another, and, since gravity plays a specific role in mortarless construction, their position in relation to the vertical of the place. In order to achieve the maximum degree of generality, one must choose a wall that is sloping rather than vertical, and a vault axis direction that is not horizontal, hence the term "sloping". Finally, the axis of the vault can have any direction relative to the wall except one that belongs to a vertical plane perpendicular to the wall, in which case the vault is no longer "sloping". In order for the vault to be fully determined, a directrix must also be given for the cylinder, which involves considering two situations. Either the curve on the face wall is given, if the architect wants the visible arch, or "face arch", to be semi-circular for the façade to be homogeneous; or, on the contrary, the cross-section of the cylinder (i.e. the section of the cylinder that is perpendicular to the axis of the vault) is fixed and the corresponding arch is said to be orthogonal. As Frézier wrote: "It is up to the architect to know whether he would rather have regularity on the outer face than inside, or whether he must cast the irregularity onto the face in order to make the inside of the vault more beautiful" [6, t. 2, p. 172].

The drawing is more or less difficult depending on whether the initial given is the orthogonal arch or the face arch. For the cutting of a voussoir, the true size of the orthogonal arch must first be determined as the initial block of stone will be pared down on this basis. Thus the bed joint surfaces will suffice to define the voussoir completely. Knowing the face arch is therefore not necessary for carving, and the tracing is very much simplified if we suppose that the orthogonal arch is given. Conversely, if the face arch is the initial given, the author must begin by indicating the construction of the orthogonal arch.

The specificity of the dry-stone piece under study is that everything is "skewed": the object itself has no horizontal or vertical plane on which one might "naturally" rely for drawing. Prior to any geometric construction, authors are therefore confronted with the problem of choosing a coordinate or reference system. More specifically, this drawing involves changing coordinates, i.e. going from one set of coordinates that express the givens of the problem (a horizontal and a vertical plane) to a set of coordinates allowing the panels to be drawn easily (a plane perpendicular to the vault axis and a plane parallel to it). Now the notion of changing coordinates is a complex notion, which Gaspard Monge, himself, did not explicitly develop in his descriptive geometry course.[2] Furthermore, the formalisation and use of coordinate change in stereotomy gave rise to measured opposition between the two main teachers of descriptive geometry in France in the 19th Century, Théodore Olivier and Jules Maillard de la Gournerie. It is therefore hardly surprising that the authors of stone carving treatises experienced some difficulty when faced with its use.

Geometrical Principles and Choice of Coordinates

Given the underlying difficulty, the various authors chose radically different geometrical principles for their graphical constructs. Philibert de l'Orme restricts the problem in two ways. First he supposes that the wall is vertical, draws the sloping vault starting from the orthogonal

[2] About the creation of the descriptive geometry by Gaspard Monge during the French Revolution, see [15].

arch (a semi-circle arch on his blueprint) and does not represent the face arch. The drawing is much simpler in this situation, as we saw previously, but is surprising for a "cellar slope", which is the title the author has given to this illustration. Indeed for a vault that covers a subsidiary space, priority should be given to the arch that appears on the face wall. The fact that Philibert de l'Orme makes such an "anti-architectural" choice certainly goes to show the difficulty he encountered in presenting (and possibly executing) the drawing of a sloping vault, starting from the face arch. The four other authors propose solutions both for the sloping wall and a regular face arch.

Philibert de l'Orme

Philibert de l'Orme and Frézier basically choose the same principle (though with different outcomes), consisting in bringing the problem back to one of drawing a horizontal vault adjoining a wall of any slope. But whether the wall is sloping and the vault horizontal, or the wall vertical and the vault sloping, is not equivalent. Rotating the entire vault suffices, it is true, to go from one position to the other. It is no doubt how Philibert de l'Orme reasoned although he does not say a word on the subject.

Judging the first situation more practical, he very naturally used this approach. In the case of an orthogonal slope, there is no disadvantage in supposing that we have a horizontal vault against a sloping wall. But obliqueness here creates an unavoidable difficulty and the reasoning becomes wrong. If one rotates Philibert de l'Orme's vault drawing from a sloping to a horizontal position, this rotation can only take place around an axis that is perpendicular to the vertical wall against which the vault is resting, i.e. an axis that is parallel to the ground line on the explicative drawing. But if the vault axis is made horizontal by performing such a rotation, the oblique section of the face wall will not be the same before and after the rotation. This problem does not arise in the case of an orthogonal slope because the rotation axis is parallel to the horizontal lines of the wall plane. Thus if we want to commute the problem from one involving a sloping vault to one involving a horizontal vault resting against a sloping wall (which is still possible), we must consider the modification of the oblique section, as a result of rotating the object [cf. Fig. 13.4]. This is in no way approached in the work of Philibert de l'Orme, who draws his layout using as reference planes the plane perpendicular to the frontal plane, which contains the vault axis, and a plane perpendicular to this axis. But the vault is only represented using this reference system, where neither the vertical, nor the slant angle with respect to a horizontal plane are given.

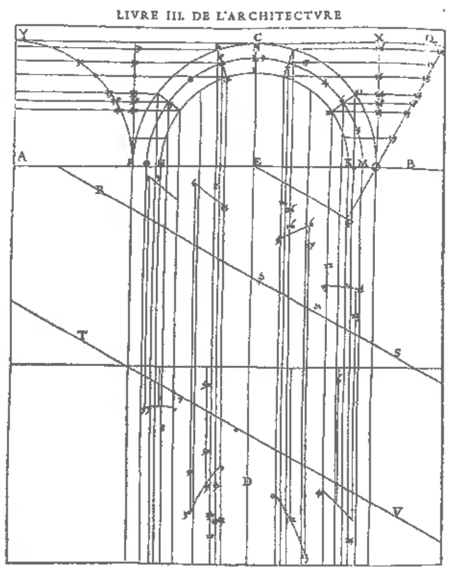

Fig. 13.2: De L'Orme's blueprint, in [10], Fol. 62, v°.

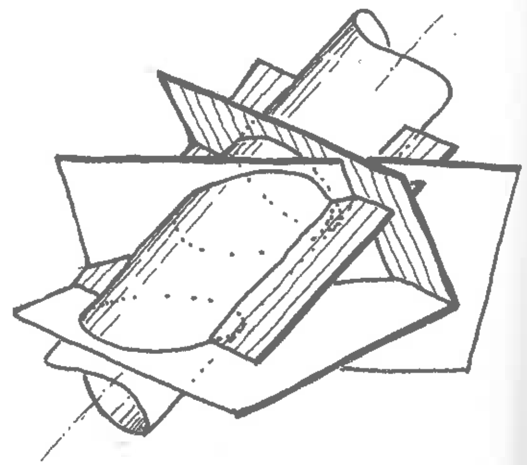

Fig. 13.3: De L'Orme's reference planes (Author's drawing).

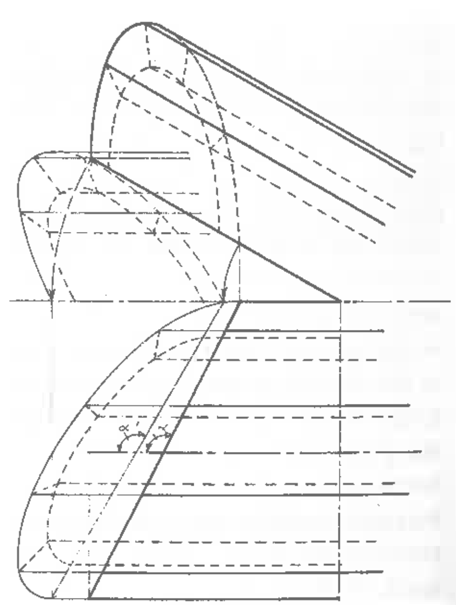

Fig. 13.4: De L'Orme's mistake (Author's drawing).

Frézier

Two centuries later, Frézier again adopts Philibert de l'Orme's principle but this time, without making the same mistake. With his usual thoroughness, Frézier considers all (or nearly all) the possible cases of vaults in his treatise: horizontal or sloping, orthogonal or oblique, adjoining a vertical or sloping wall, with a given orthogonal arch or face arch. Given the number of situations studied, and in order to derive a methodological approach to the problem rather than just display a juxtaposition of drawings, he systematically chooses to bring the problem back to the "previous problem". Frézier certainly exhibits great rigour and offers a very homogeneous presentation. Simpler cases can be deduced from more complex cases without changing the method in the slightest. In the blueprint of the orthogonal slope adjoining a sloping wall, for instance, if the slope angle is zero, we revert to the blueprint of the horizontal vault against a sloping wall. But, as we saw previously in the case of the layout given by Philibert de l'Orme, when the constraints accumulate—sloping vault, then a sloping vault against a sloping wall, keeping the initial principle and wanting to bring the problem back to the "previous problem" can present disadvantages.

In order to avoid the mistake of his illustrious predecessor, Frézier chooses other reference planes. He draws the profile view on a vertical plane that is parallel to the axis of the vault. This layout seems optimal given the problem treated because this view is essential. It is therefore natural, and more comfortable for the reader, to have this profile view appear as a main frontal view, i.e. horizontal on the page, and not by changing the frontal plane to a somewhat baroque position for the reader, as in Derand, de La Rue or... Hachette.[3] The second step is not so convincing: it consists in determining the projection of the vault onto what Frézier calls the "ramp plane", i.e. the plane (N on the fig. 13.6) from which the vault is erected, and folding it onto the profile plane of the vault in order for the projection to appear according to its true size. As mentioned earlier, when the given is the orthogonal arch, this plane is perpendicular to the frontal plane and the process involves performing what we now call a change in horizontal plane. In the new coordinate system, the layout is that of a horizontal vault against a sloping wall. The face arch is folded onto the new "horizontal" plane for it to appear according to its true size. But when the given is

[3] Jean Nicolas Pierre Hachette (1769–1834) succeeded to Monge, in 1797, in teaching descriptive geometry at the Ecole polytechnique.

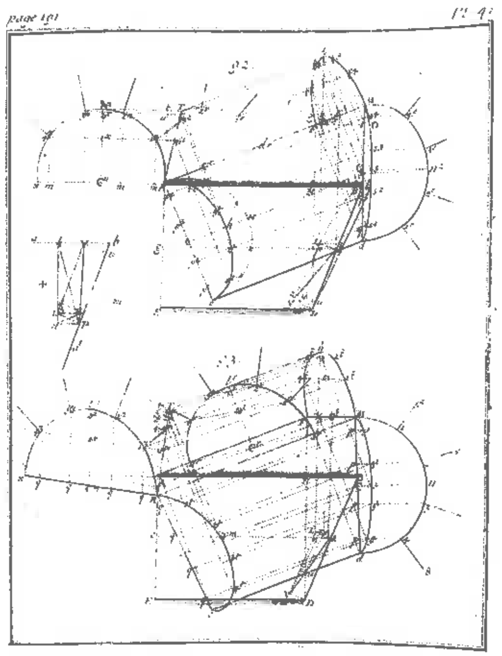

Fig. 13.5: Frézier's blueprint, in [6], pl. 41.

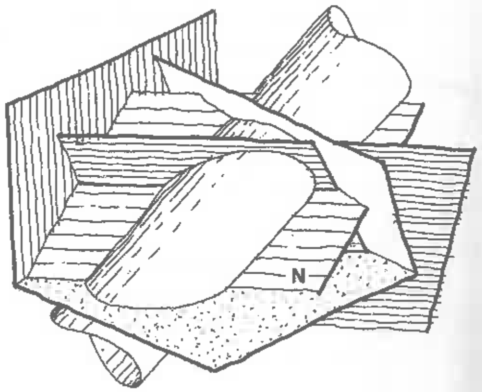

Fig. 13.6: Frézier's reference planes (Author's drawing).

the face arch, the ramp plane can be any plane. Frézier explains this very well: "if we project the vault onto this plane, vertical lines will slope relative to it... Thus... we shall take the ramp plane as the horizontal and the vertical plane passing through the axis as the incline relative to the horizon". But we cannot think of this operation as a change in horizontal plane without losing the concept of verticality, and hence gravity.

Desargues

Desargues adopts another view point and the graphical construct he proposes, though very simple in principle, is rather confusing. Since the main problem is to derive a figure drawn on the plane perpendicular to the vault axis from a figure drawn on the face wall plane, the auxiliary plane that needs to be introduced to go back and forth easily between these two figures is a plane that is simultaneously perpendicular to the two preceding planes. Among this family of planes, it is then quite natural to choose the plane that contains "the axle", or, according to Arguesian terminology, the vault axis [Fig. 13.8]. This is

the plane that the author calls the "sub-axle plane" and we can assume that Desargues' figures (apart from the first, which is a drawing of the jambs and not the voussoirs) are drawn in this plane. If we suppose the face arch is given, the first step in the construction, which is in fact the essential step, consists in deriving the orthogonal arch, that is to say the cross-section of the vault. With respect to the sub-axle plane, which is taken as a reference plane, going from the orthogonal arch to the face arch is an operation similar to the change in frontal plane in descriptive geometry. This coordinate system makes the construction of the orthogonal arch from the face arch and *vice versa* easier but requires an initial layout in order to present the assumptions of the problem.[4] The Arguesian choice is therefore very clever, but by taking a three-dimensional coordinate system that is intrinsically linked to the object, Desargues deprives his method of the universal quality he was aiming for. He also deprives himself of the possibility of treating architectural objects of a radically different nature from that of the object chosen, either because they comprise curved surface intersections, which would be the case if the opening were made in a cylindrical or conical wall, or because they present surfaces with a double-curvature radius such as Marseille *arrière-voussures*, squinches, domes or spiral-staircase strings. In addition, by choosing a coordinate system where all reference to gravity has disappeared, his opuscule becomes far less accessible.

As La Gournerie noted: "When considering an abstract system of lines and surfaces, one can suppose its transposition one way or another without any problem, but when dealing with an architectural work, such as a vault or a staircase, it is better to consider it in its natural position and study it on its plane and elevation. If it is made to rotate in space, if it is projected onto planes none of which are horizontal, the mind experiences some difficulty in imagining it, and the layouts, while just as simple from the geometrical point of view, become more difficult to grasp" [8, p. 45]. By (implicitly) taking the "sub-axle plane" as a reference plane, Desargues destabilises his readers, forcing them to reason on the basis of a plan that is neither vertical nor horizontal in space. He places his readers on a plane that rolls and pitches simultaneously, or thinks about the vault as if it were a satellite under zero gravity, which comes to the same thing [Fig. 13.8]. If, from the purely geometrical

[4] The component element of this "initial layout" is given by Bosse [1] in the form of 14 illustrations with comments.

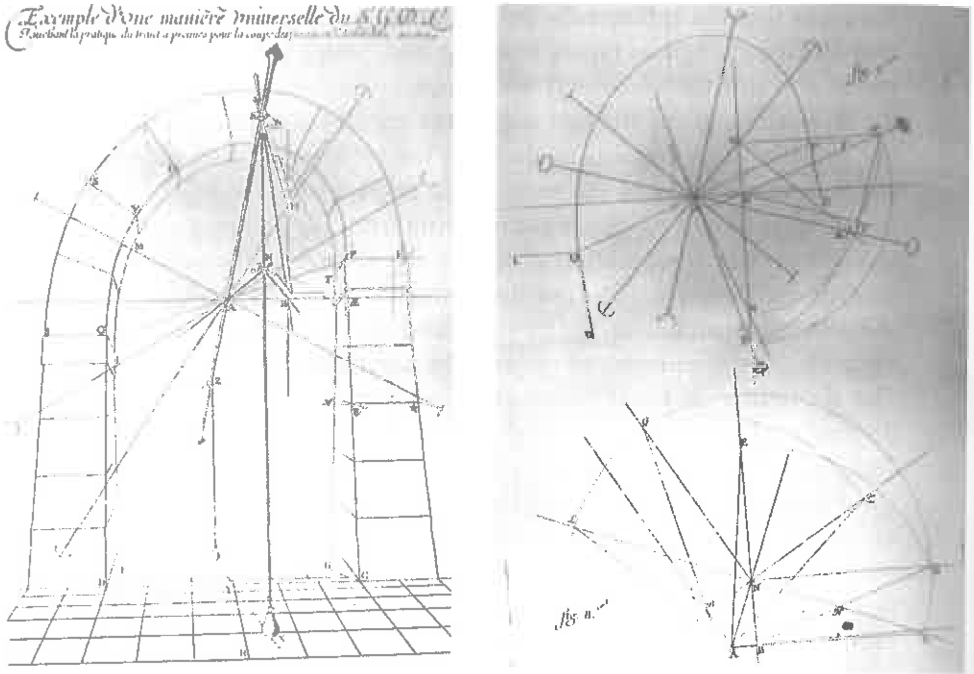

Fig. 13.7: Desargues' blueprints, in [5], pl. 1 and 2.

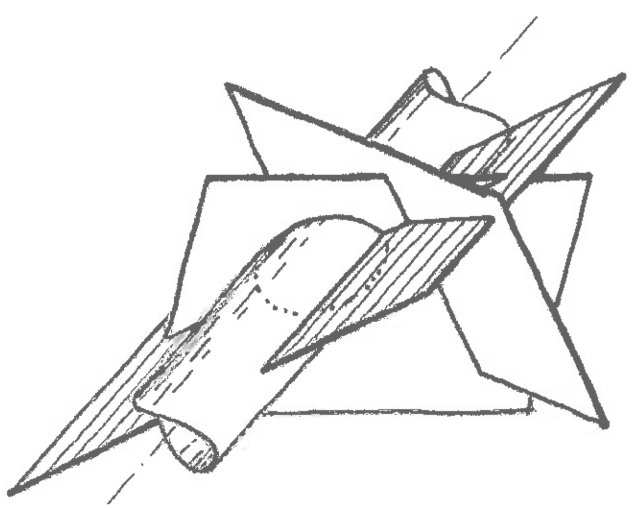

Fig. 13.8: Desargues' reference planes (Author's drawing).

point of view, when two planes, A and B, are superimposed, the result is the same whether one considers that plane A is fixed and plane B is folded over it or B is fixed and A is folded over it, in the case of an object with a mass, one of the two operations can be far more difficult to understand than the other. For example, if one of the planes, A shall we say, is horizontal in physical space and B has any direction, the first operation will be much easier to imagine than the second.

Now, in the case of Desargues' manual, the situation is complicated twice over. First of all because the game is played not with two planes but three or four planes. Secondly, because the "optimal" reading grid, the one that can reflect the geometrical operations in the simplest way possible, is not the same for the first figure, where Desargues determines the relative position of the four axes he has introduced, as for the following figures. Indeed, for the first figure, it is better to consider the plane of the face wall as "fixed", while for the following figures, it is simpler to suppose, as described above, that the sub-axle plane is the plane of reference. One can certainly object that these remarks are quite subjective, that each of us can find this or that situation more comfortable or "natural", and that the author, by giving no further details on this issue gives the reader the freedom to choose the reading grid that suits him best. In any event, being a good geometer, Desargues probably felt free of any constraints that were not strictly geometrical and one cannot doubt the ease with which Desargues went from one plane to the other or his mastery of the situation. But gravity remains the most shared of all things and, in a presentation that aims to be explicitly didactic, making total abstraction of it is not necessarily the best way to convince the reader.

Jousse

Jousse, like Derand or Frézier, clearly separates the various component steps and the case of the vertical wall from that of the sloping wall. Jousse chooses as a reference plane a horizontal plane and a vertical plane perpendicular to the horizontal projection of the vault axis. His layout consists in determining the orthogonal arch from the vertical section of the vault cylinder. If the face wall is vertical, this construct is satisfactory.

But, if the wall is sloping, it becomes arbitrary. Indeed, Jousse does not project the object on this vertical plane, but considers the intersection of the vault with this plane and uses this vertical arch to go from

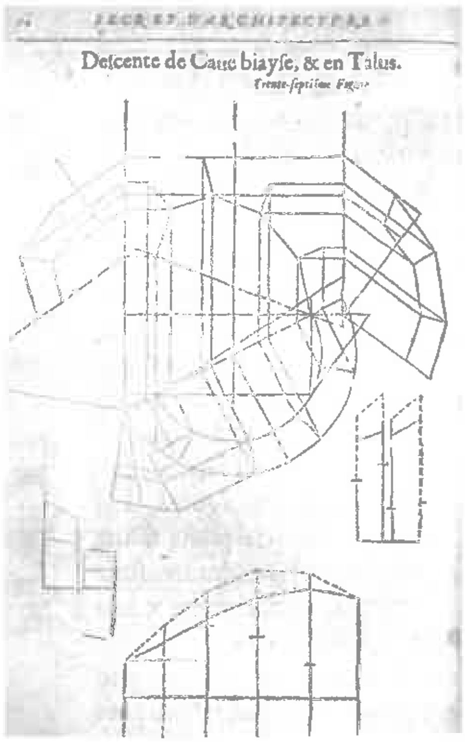

Fig. 13.9: Jousse's blueprint, in [7], p. 74.

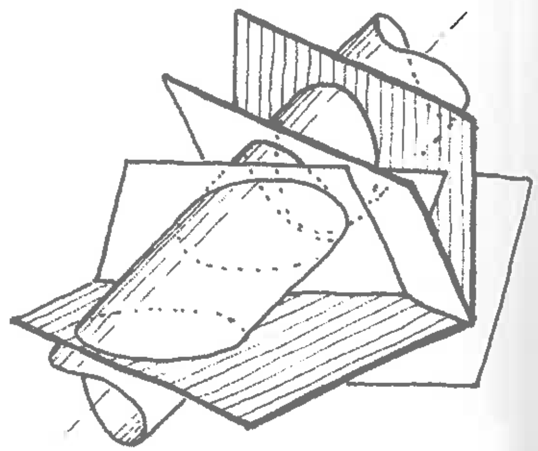

Fig. 13.10: Jousse's reference planes (Author's drawing).

the face arch to the orthogonal arch [Fig. 13.10]. The vault is seen as an unlimited cylinder with a directrix lying in a vertical plane including a horizontal line from the face wall. Although this layout presents the advantage of a tangential joining of the face arch on the edges of the jambs, this is probably not the reason for choosing this layout. While Jousse makes no comment to this effect, the construction he presents is rather general: it is valid for any arch of the face wall and is also "reversible". It can be read going from the orthogonal arch to the face arch or vice versa, which offers the architect a degree of freedom Philibert de l'Orme does not provide. Let us add also that this blueprint is coherent with the previous drawings. For example, if the slope is equal to zero, we obtain the blueprint given for the sloping horizontal vault.

Derand

Derand devotes eight chapters and thirty-five pages to the study of sloping vaults, "a type of vault among the most difficult that art displays"

[4, p. 48]. He proposes three different solutions for a semi-circle arch on the face wall and examines the case where the orthogonal arch is semi-circular. Derand then reverts to the three solutions previously presented, but assuming that the wall is a sloping wall, presents the construction of the sloping vault against a sloping wall for any rampant arch on the face wall, and finishes his study with the squaring method. Saying that Derand feels at ease with the subject, perceives all its wealth and happily juggles with all the different solutions would be an understatement. In his first method, Derand takes a coordinate system comprising a horizontal plane and a vertical plane parallel to the vault axis [Fig. 13.12]; in the second, he keeps the same coordinate system but projects the vault obliquely onto the vertical plane; in the third, he performs the same change in horizontal plane as Philibert de l'Orme. The new horizontal plane contains the vault axis and remains perpendicular to the given vertical plane. But, unlike with de l'Orme, the old horizontal plane of the vault is conserved, and both the old and new planes are superimposed in the same view.

While Derand's first illustration seems close to a descriptive geometry blueprint [Fig. 13.11], the other two are very different from it both in the form and spirit of their construction. In the second illustration, Derand replaces the profile view of the vault by a very special axonometry, where horizontal lines are represented by lines that are parallel to the vault generator lines and where the segments of lines parallel to the vault axis are projected according to their true size. The third method consists in separating the information supplied by the "axonometry" of the previous blueprint on two separate drawings. On an outlined profile, Derand only keeps the first part of the axonometry, which is very easy to draw and sufficient to deduce the orthogonal arch.

Continuing with the systematic study of the sloping vault, Derand provides the drawing of such a vault when the orthogonal arch is semi-circular. This results, on the face wall, in a "depressed rampant arch, the effect of which can easily be visualized when it is well done" [4, p. 61]. Adopting, for this illustration, the principle previously followed in the first of the three solutions, he rapidly draws the blueprint by simply inverting the order of the operations. Finally, Derand considers the case of a sloping wall. Having previously shown his ability, we are therefore surprised, on this occasion, to see him make a blatant mistake. Indeed, he uses the same three methods previously developed for a vertical wall. He has no problem generalizing the first one.

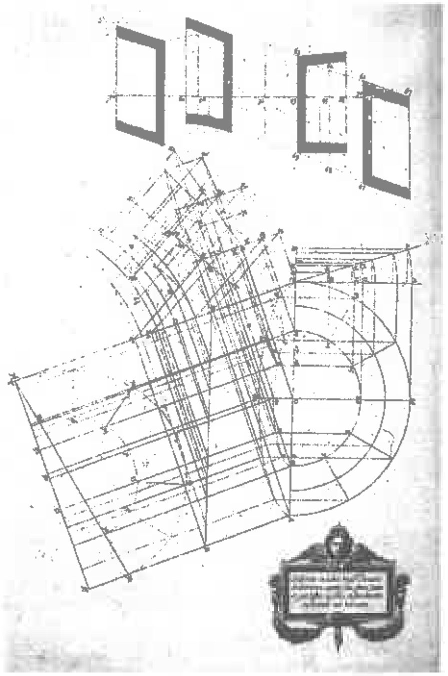

Fig. 13.11: Derand's blueprint, in [4], p. 66.

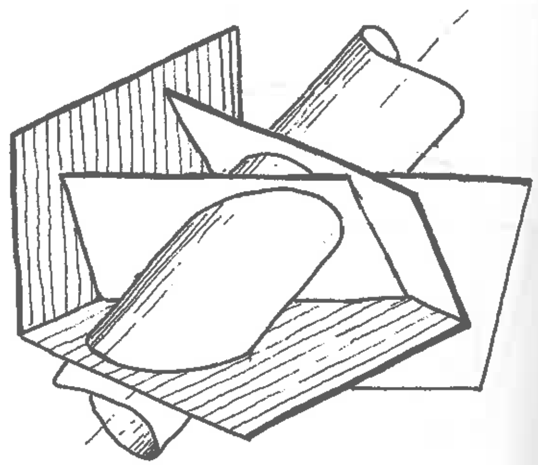

Fig. 13.12: Derand's reference planes (Author's drawing).

Derand adds a profile view, adjacent to the semi-circle arch folded over the face wall, on which he represents the angle of the wall. Unlike Jousse, he considers the semi-circle arch on the face wall. This profile view enables him to determine the horizontal projection of the face arch, to deduce its frontal view and finish the blueprint as for the vertical wall. But Derand is less successful when he attempts to apply the two other methods developed in the previous chapters. While, with some measure of good will, it is possible to interpret the diagram as an axonometry if the wall is vertical, when the wall is sloping, the construction given no longer corresponds to an oblique projection of the vault onto a plane, and the layout of the orthogonal arch is wrong. On Derand's blueprint, the error is minimal and even below the precision of the layout, but if the slant angle is increased, the error becomes significant. The rest of his reasoning is no less wrong, for the same reason. Derand indeed pursues his construction as he did in the case of the vertical wall. He performs a change in horizontal plane with the result that individual edges, but not the bed joint surfaces, are projected according to their true size onto the new reference plane.

"I must inform you...that these three methods for drawing sloping vaults adjoining a sloping wall...were performed using the same measurements, with a view that the resulting panels be equal in every way and that the same effect justify such good practices, which will therefore constitute proof for one another" [4, p. 74], concludes Derand. This justification is a little quick, first of all because the last two methods presented are coherent with respect to each other and simultaneously wrong, and secondly, because the error can only be noticed graphically, both in the case of the panels and the orthogonal arch, if the slant is substantial.

In the last chapter devoted to this subject (other than the squaring method), Derand studies the drawing of any rampant arch on the face wall and places himself exactly in the situation considered by Desargues. He rightly uses the first method presented for the semi-circle arch. Since this method never calls for the use of special properties of the face arch, it can be used without modification. It is, however, surprising that Derand adds a chapter to present a situation that is not really more general. The vault layout is correct because of this, but Derand makes a slight mistake in the simplest part of the layout, which concerns the jambs.[5]

There are as many methods of approaching the problem and different possible choices of reference planes as there are authors. This reveals the difficulty the authors experienced in approaching the problem. To draw the blueprint of a sloping vault, one of the reference planes must be parallel to the vault axis in order that the segments plotted from its generator lines appear according to their true size. The solutions listed above all consider the "reasonable" choices, given the premises of the problem. But this plurality of approaches shows that the basic principle, which consists in taking a horizontal plane and a vertical plane, was not inherently and automatically obvious before Monge. The latter nonetheless declared in his course at the Ecole Normale, that "artists...generally suppose that, of the two projection plans, one is horizontal and the other vertical" [11 p. 315]. Whatever Monge had to say, this custom was only well established before he began teaching when the object was itself naturally linked to such a coordinate system. The principle of the horizontal plane-vertical plane reference system

[5] Given the sloping wall, the vertical part of the passage wall is not, when projected horizontally, perpendicular to a horizontal line of the wall. Indeed, Frézier points this out in his work, [6, t. 2, pp. 187–188].

did not even prevail after Derand's treatise. Nor did it totally prevail for Derand himself, who somehow gives the impression of having used it by serendipity. What did he look for after having presented his first method? No doubt to present a blueprint with less lines to draw, which was a very essential preoccupation for masons. But this led Derand to make a blatant mistake, precisely because he ventured to draw layouts without any natural reference system. De la Rue makes use of the first two methods given by Derand, without particular preference for the one derived from a natural reference system. Finally, Frézier, who knew Derand's treatise thoroughly and presented the principle of the double projection in the first tome of his own treatise, proposes a different solution that is not quicker to draw and far more difficult to understand.

Indeed, Frézier writes in a paragraph of the first tome entitled *de l'Arrangement des desseins dans l'Epure*: "Although it is more natural to separate each drawing, it is nonetheless true that this simplicity of object does not indicate so well the relationship between the lines, and that it is therefore not as convenient as gathering, and some times even mixing the layout, profile and elevation: the arrangement of their situation, close to, within, above, below or next to each other should however be viewed as arbitrary" [6, t. 1, p. 272].

Conclusion

A certain number of conclusions may be drawn from this comparative study.

The problem of two-dimensional representation of three-dimensional objects

One of the elements all these drawings and explanations have in common is the total—or quasi total—absence of an object in space. The constructions are drawn in the layout, without the points, curves or surfaces from space seeming to intervene. They are never named, except by Desargues, at the very beginning of his construction. Hachette distinguishes two steps in the resolution of a three-dimensional geometrical problem. The first concerns the resolution of the problem by three-dimensional geometry, and the second is the graphical rendering of the former. In the illustrations studied, and more generally in stone carving treatises, the first step is totally absent from explicative texts.

In descriptive geometry, on the contrary, accompanying texts give spatial geometrical constructions that are graphically translated into drawings. Descriptive geometry is precisely the language that is appropriate for describing space. As long as such a language has not been defined with sufficient precision by enunciating a few basic rules, certain mental operations remain untranslatable. This results in a distance, always surprising for present day readers, between the extraordinary ease the authors of such treatises seem to have for manipulating complex volumes and surfaces, and their capacity to clearly present their solutions. The explanations boil down to a succession of directions without justification: "trace this line, carry over this distance, etc..." without ever giving the reason for the tracing or the mental operation the reader ought to perform. The modern reading of these texts is summed up by the recurrent question "what is he doing"? Clearly, such questioning derives from a state of mind that is directly opposed to that prevailing among the various authors, who never sought to justify their drawings.

Transmission of know-how

In stone carving treatises, two different problems tend to interfere. The first is of a geometrical nature and involves determining the true size of the voussoir joint surfaces. The second is of a didactic nature: the reader must understand the drawing proposed in order to be able to reproduce it, or be in a position to produce a drawing to solve a precise problem he is faced with. Should a stone carving treatise be a collection of drawings for master masons? Philibert de l'Orme and Jousse appear to acquiesce. However, the real-size layout the master mason produces is one thing, and a drawing explaining how to make a drawing for a master mason is another. Confusion between these two steps may lead at worst to errors and at best to difficulties in reading and communication. Philibert de l'Orme seems to be aware of the problem he is facing. He explicitly admits it at the end of his commentary on the illustration of sloping vaults, when he writes that "other [drawings] might be made that are difficult to carry out...but apart from it being a head breaking exercise to reflect on them and represent them, I would also fear that few people in this world might sink their teeth into them and the simple demonstration I might make of them" [10, p. 62]. It is therefore not totally excluded that the drawing necessary for oblique vaults, when the orthogonal arch is to be determined, is beyond the pedagogical skills of the author, and even possibly his practical competence. According to Philibert de l'Orme, all cannot be presented in written form and a treatise cannot totally replace teaching by oral transmission and practical demonstration.

The drawings studied generally meet their main objective satisfactorily, in other words to give the exact geometrical construct of the joint surfaces. On the other hand, the second objective, the transmission of know-how, is only very partially attained. The main reason for this failure comes from the dichotomy, which we have just examined, between representation and geometrical construction. The graphical representation of the object is either totally absent, or entirely dependent on subsequent geometrical drawings. Descriptive geometry, on the other hand, makes it possible to unite these two functions on a single drawing, bringing together the technical drawing and the pedagogical tool.

Statical considerations

Curiously, statical considerations are totally absent from all the commentaries on sloping vault drawings. However, if the obliqueness is very pronounced, independently of the sloping vault or wall, the drawings proposed by the various authors, though they remain geometrically correct, become inapplicable in practice. Voussoirs with angles that are too acute might explode under the thrust of the vault, as Rondelet points out. The authors studied do not mention this limitation and remain strictly within the realm of purely geometrical considerations.

The strong dichotomy between the geometrical and statical approach to the problems inherent to dry-stone vault construction lasted a long time. One has to wait for Rondelet's *Traité théorique et pratique de l'art de bâtir*[6] to find a work that tackles the problems linked to mortarless architecture from a geometrical, statical and economical point of view, and expounding on the differences in the materials used, the quality of the cement and stones, etc.

[6] Jean-Baptiste Rondelet (1743–1829), a student of Blondel, became Soufflot's successor on the Sainte-Geneviève Church building site in Paris. He erected the dome and took care of its transformation into the Panthéon. Rondelet was a member of the *Commission des Travaux Publics*, who decided the creation of the Ecole polytechnique.

The issue of geometry

Placing geometry at the heart of architectural training was a wish Alberti had already formulated. But for Renaissance architects, the real need for geometrical knowledge in everyday practice remains very modest. Hence Alberti's wish only becomes truly meaningful and justified at very specific steps of architectural creation, drawing remaining the best case in point. Stereotomy, "that which is most refined and artistic in architecture", following a word of Claude Perrault, is the place where architecture meets geometry. From this meeting will arise a new geometrical theory, descriptive geometry, as well as the figure of the modern architect. Philibert de l'Orme clearly expresses the need for this marriage between geometry and architecture, which his work inaugurates to a certain extent. He insists on his goal, which is "to join the practice of architecture with the theory of the said Euclid" [10, p. 62]. But given the stakes at hand, the steps leading to this "wedding" were rather violent, jolted and conflict ridden.

The most symptomatic episode among these tensions was the quarrel opposing Curabelle, one of the most famous master masons of his time and Girard Desargues. The essence of this quarrel does not bear so much on the content of the stone cutting manual proposed by the Lyon geometer as on the manner by which one might be sure of its legitimacy. For Curabelle, feasibility is of course the criterion whereas for Desargues, the only thing that matters is the correctness of the geometrical reasoning. By virtue of this opposition, the entire status of the blueprint drawing is being questioned. If we accept, along with Curabelle, that the drawing can only be deemed worthy and validated by execution, the master mason remains the keystone of the building site. Preliminary drawings are certainly necessary but have no autonomy and cannot be dissociated from the construction they make possible. They are but a first phase, a first step of one and the same process of production.

If, on the other hand, a drawing can, as Desargues claims, have its very own legitimacy, if one can convince oneself of its correctness by purely theoretical considerations independent of any concrete actions, if geometrical reasoning rather than experience is what allows the finding of the optimal lines, then the status of the drawing itself is modified, and so is the status of the author and builder. The master mason is therefore deprived of part of his role and power. Desargues expresses this quite brutally: "just as Doctors...do not attend the school or receive the teachings of apothecaries... Geometers... neither attend the school nor receive the teachings of Masons; quite to the contrary, Masons... attend the school and receive the teachings of Geometers, which means that Geometers are the masters and Masons the disciples".[7] What clearly emerges from the polemic writing of Desargues is the claim that theory prevails over practice.

The camps are therefore well defined among stone carving aficionados. On one side, practitioners such as Jousse, Derand, Curabelle or de La Rue defend the master mason profession throughout their work, and on the other side, theoreticians such as Philibert de l'Orme, Desargues, Blondel, de la Hire or Frézier attempt, to various extents and with more or less success, to integrate academic geometry as a lever for transforming a construction technique and changing power relationships between the various social actors.

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