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El Arte de la Piedra Teoría y Práctica de la Cantería

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Coordinación editorial de la colección: Juan Roldán Martín
CEU Ediciones
Julián Romea 18, 28003 Madrid
Teléfono: 91 514 05 73, Fax: 91 514 04 30
Correo electrónico: publicaciones@ceu.es
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www.uspceu.es

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From the Stone Carver's Techniques to Descriptive Geometry

Joël Sakarovitch
Paris Descartes University
School of Architecture Paris-Malaquais

According to a clearly oversimplified though not totally unjustified view, the architect is first and foremost the person that can conceive and graphically translate a building project. The graphical techniques used in this process are so strongly associated with architectural function that in everyday language, the use of a triptych section-plane-elevation representation of a building tends to be referred to as an «architect's drawing». Such a representation is said to be «geometral» according to the terminology specific to this art. Geometral representation allows the various building trades in charge of erecting the building to understand and then proceed to build using all the necessary information in the graphical documents, including measurements of the various parts of the building. In order for the building trades to comply rigorously with the project imagined by the architect, both overall and in every detail, the graphical language used should be devoid of any ambiguity, which explains why representations with a conical perspective (analogous to a photograph) or an oblique parallel (cavalier) projection (used for instance in instructions for the assembly of objects sold as separate parts) are only used to supplement rather than substitute for geometral representation.

The idea that the architect is a plan producer is relatively recent in the history of architecture. The first drawings clearly displaying articulation between two views are not to be found before the XIVth century and remain very exceptional in the XVth century. A drawing for the Milan cathedral, attributed to Antonio di Vincenzo and dated 1389, provides the first example where two distinct views, a ground plan and partial section,

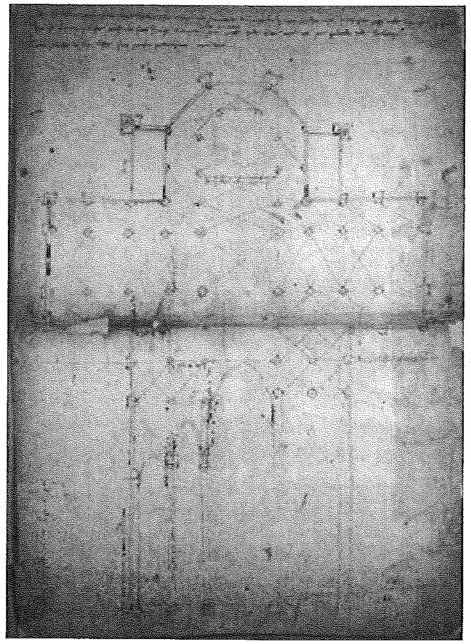


Fig. 1: Antonio di Vincenzo. drawing for the Milan cathedral, 1389. The first example where two distinct views, a ground plan and partial section, are clearly articulated with each other

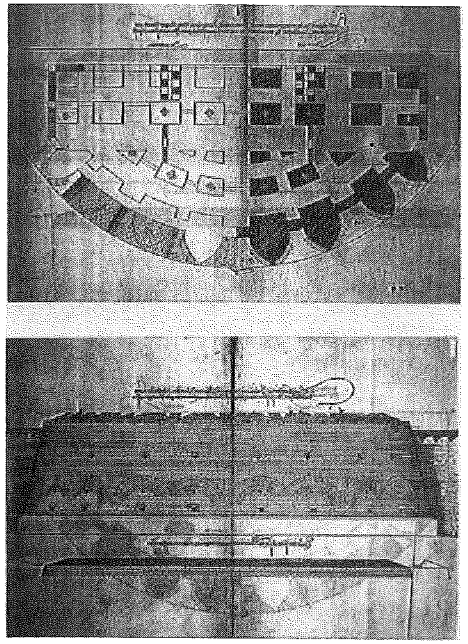


Fig. 2: Dürer, Ground track and elevation of a bastion *Etliche underricht, zu befestigung der Stett, Schloss, und flecken*, 1527.The first drawing of a building using section-plane-elevation representation

are clearly articulated with each other[1] (fig. 1). The majority of known architectural drawings from that time remain desperately «flat» and correspond to what we would call today a projection onto one plane. This is not to say that the designer did not conceive of the object in space but the latter being built by *opus in mente conceptum*, the crucial drawing step prior to construction was not indispensable given the worksite organization conditions, the constant presence of the architect, the degree of freedom left to masons during the work, etc. *Principalis artifex* during Antiquity and the Roman period, there is nonetheless evidence for graphical production on the part of architects for over four thousand years[2]. But the tracing of a ground track, a vault profile, an elevation drawing (or even a ground plan and elevation) fail to constitute *ipso facto* an geometral representation. Indeed the latter not only requires that the three views be traced on the same scale, but also that they be thought of and articulated in relation

1 For a project to build the bell tower of the Fribourg-en-Brisgau cathedral at the beginning of the 14th century, the ground plan and elevation are drawn in direct correspondence;. It is nonetheless worth noting that the first elements of double-projection representation of certain architectural details appear in the Villard de Honnecourt's *Carnet* (middle of the XIII[th] century). On this subject, see J. Sakarovitch, *Epures d'architecture, de la coupe des pierres à la géométrie descriptive, XVI-XIX siècles*, Basel, Birkhäuser, 1998, chap. 1.

2 The oldest testimony comes from a Mesopotamian statue, known as the Seated statue of Gudea with architectural plan, where Gudea (governor of the city of Lagash) is holding on his lap a plan thought to be for the temple of Ningirsu (Louvre Museum, circa 2200 BC). One of the most famous papyrus plans is that of the Ramses IV tomb (middle of XII[th] century BC),which presents an extremely rare advantage for this particular period in that it corresponds to a building that still exists.

to one another. Only then can one truly consider that a graphical spatial representation technique has been devised to meet the modern needs of architects and engineers. While the underlying principal in geometral representation remains simple and only requires the use of relatively simple geometric tools (orthogonal projection essentially), this spatial representation mode is at the same time very abstract, which explains the additional use of more readily understandable representations. This also explains the rather mysterious and esoteric atmosphere surrounding the know-how of corporations using drawings based on axonometric projections.

While geometral representation is nowadays associated with architectural drawing, throughout the Middle Ages and the Renaissance, it was first and foremost associated with the drawings of stone carvers. «*Hence the necessity for anyone wishing to address the study of proportions to have understood how to make measurements and thoroughly grasped how any object must be laid out in its ground plane and elevated, according to the method stone carvers use in their daily practice*» writes Dürer[3], for example. In order to show that geometral representation indeed originated from stone carving techniques and understand how geometral representation was literally «built» from such techniques, it is necessary to present them rapidly.

Stone Carving Techniques

There are three major stone carving techniques: rough casting, squaring and template cutting. They can be compared using the same dry-stone architectural structure[4], namely a straight door in a round tower (Fig. 3), which can be achieved by each of these methods.

Rough casting consists in carving the stones once they have been fitted in the vault or arch. The stones are roughly cut on the ground and only given their exact shape once they are in place. In the example considered, it is possible to carve the voussoirs without taking the tower into account, as if the arch belonged to a flat wall, and to remove the extra stone afterwards (Fig. 3).

Carving by squaring, also called very graphically «derobement», consists in «carving a stone... taking into account the height and depth that determine the limits of what needs to be removed, as if one stripped the imagined figure of its clothing»[5] (Fig. 4). Starting from real size tracing of the ground track and elevation, the foreman[6] traces the

3 A. Dürer, *Heirinn sind begriffen vier Bücher von menschlicher Proportion...*, Nuremberg, 1528, note to W. Pirckheimer; cf. J. Peiffer, «Dürer géomètre», introduction to the translation of the *Underweysung der messung...* de Dürer, Paris, Seuil, 1995, p. 59.

4 In dry-stone architecture, vaults and arches are composed of arch stones or voussoirs with oblique sides via which they lean on adjoining voussoirs.

5 A.-F. Frézier, *La théorie et la pratique de la coupe des pierres et de bois pour la construction des voûtes... ou traité de stéréotomie à l'usage des architectes*, Strasbourg, 1737-39, t. 1, p. 397.

6 On a stone cutting worksite, the foreman is responsible for tracing scale drawings, i.e. drawings on a scale of 1 allowing

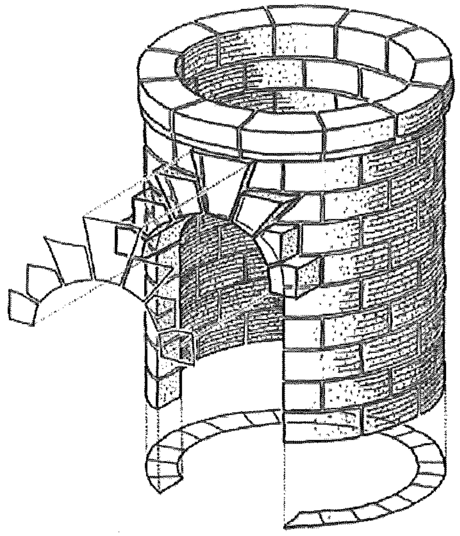


Fig. 3: A straight door in a round tower. Rough casting technique

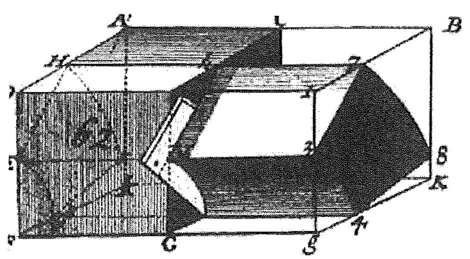


Fig. 4: Carving a voussoir by squaring

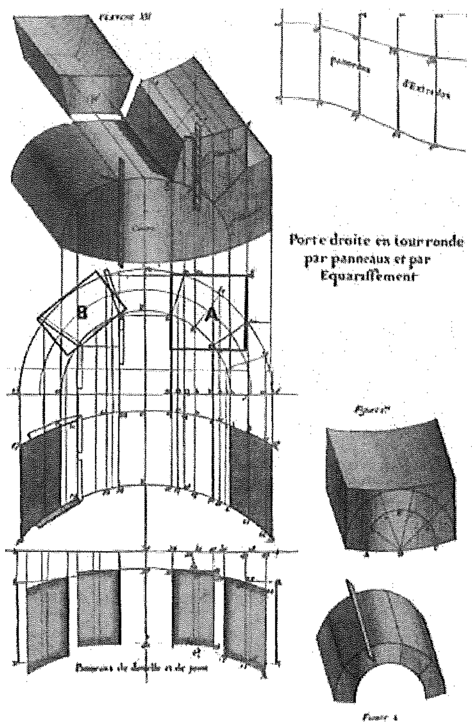


Fig. 5: Voussoirs of a straight door in a round tower, cut by squaring and template cutting

voussoir in the position it will occupy after assembly inside a rectangular parallelepiped with horizontal or vertical sides. Each vertex, or any given point, is then located by its projection onto two orthogonal faces of the envelope. In our example, in order to cut voussoir A (Fig. 5), the foreman traces out the portions of the ground track and elevation on the corresponding faces of the squared block of stone he started from. The voussoir is then cut using this tracing alone (Fig. 6). It is then a matter of cutting the stone so as to form a cylinder with vertical generator lines and director curves traced out on the horizontal planes[7], and similarly to form the cylinder with horizontal generator lines, thus allowing the desired voussoir to appear at the intersection of the two cylinders without

the shape of each voussoir to be precisely determined.

7 The term «cylinder», in accordance with its use in geometry, designates any surface defined by the set of generator lines parallel to a given direction and lying against a given curve (called the generator).

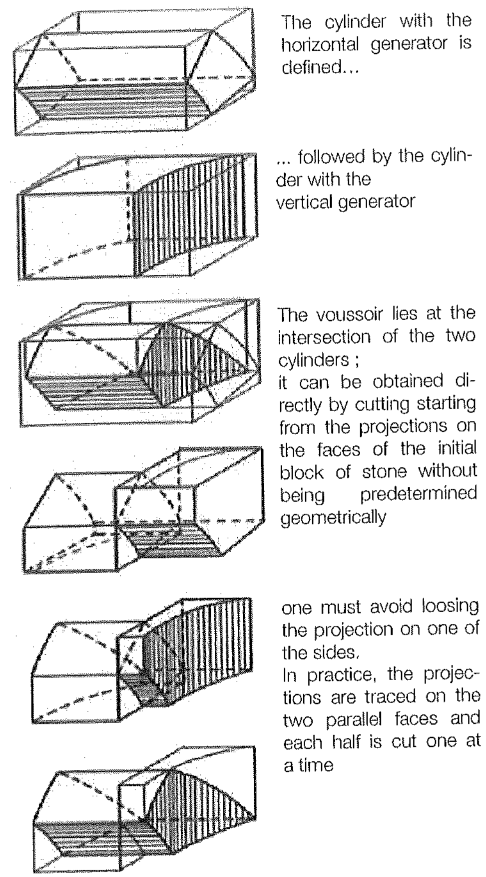


Fig. 6: Cutting of voussoir A by squaring

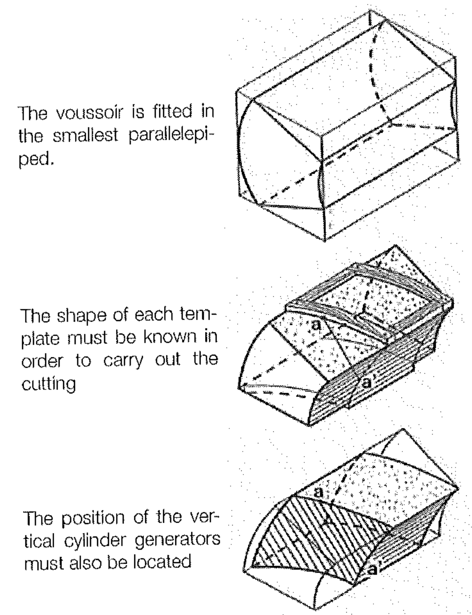


Fig. 7: Cutting of voussoir B using a template

having to predetermine its shape. Naturally, at the time of cutting, one must avoid loosing the projection on one of the sides (the vertical one for instance) by cutting the entire vertical-generator cylinder in one go. In practice, the projections are traced on the two parallel faces and each half is cut one at a time, as indicated on figure 6.

In the template method, the volume of each voussoir is determined from the surface of each of its sides. The voussoir is fitted in a minimal rectangular parallelepiped (Fig. 5, voussoir B), which now only has one plane direction parallel to one of the spatial references. In the example provided, the parallelepiped has two vertical faces and no horizontal faces. After having traced the face template as given by the elevation, the stonecutter uses it to obtains the cylinder with horizontal generators. Any reference to the ground plane-elevation reference system having been lost at this point, it is necessary to have previously determined the exact geometrical shape of each of the voussoir sides in order to complete the work. More specifically, in the example shown in figure 7, one must

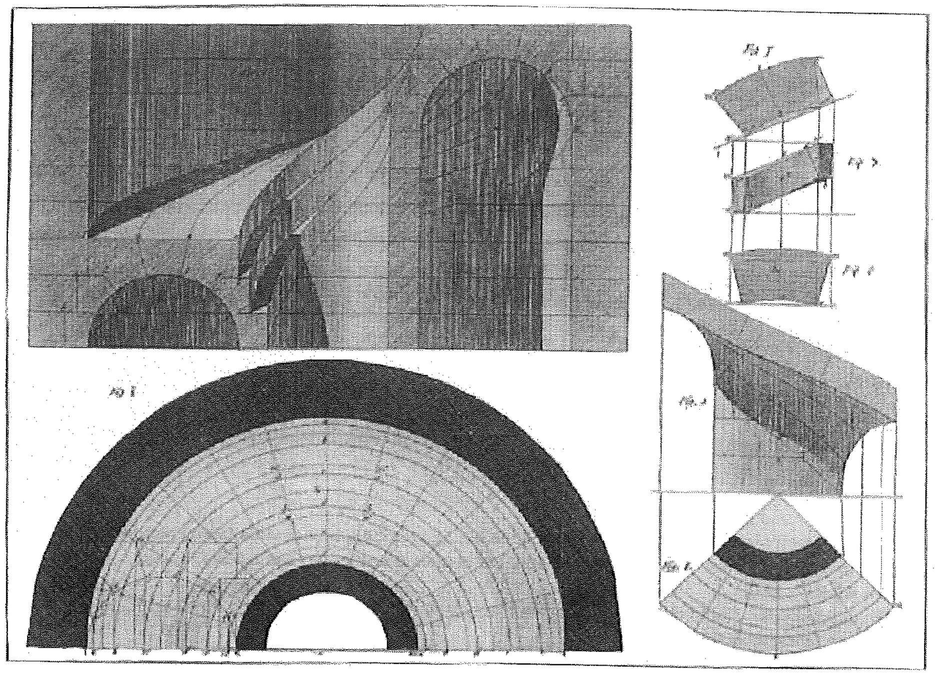


Fig. 8: Semi-circular rising vault or «vis Saint-Gilles»
From Philibert de l'Orme to Rondelet, the vis Saint-Gilles has always been considered the most difficult piece of dry stone architecture to achieve, the voussoir beds being gauche surfaces that cannot be developed and the edges having double curvature. The vis Saint-Gilles can be cut by squaring or using templates, but not by rough casting.

locate the points of intersection of the vertical generator lines of the cylinder that forms the inside of the tower with the various templates. In order to obtain a regular surface that complies with the project at the time of cutting, the stone cutter's chisel must follow the direction of the generator lines, which should therefore have been located previously (for example by points a, a' in figure 7).

A Broader Stereotomic Repertoire

The improvement of cutting techniques undoubtedly increases efficiency on the worksite. Closer to sculpture than stereotomy, rough casting has the major disadvantage of destabilizing the mortar and can therefore only be used in «dry joint» constructions. Furthermore, the stereotomic repertoire can be broadened considerably by the use of the other techniques (Fig. 8). In comparison with squaring, template cutting allows significant stone saving by reducing the squared block of stone enveloping the voussoir to a minimum (for voussoir B shown in fig. 5, roughly one third of the stone can be saved; for a *vis Saint-Gilles*, squaring generates considerable loss of stone), and above all by reducing cutting time and the cost of execution. As shown in fig. 6, cutting by squaring often leads to the cutting of surfaces that are ultimately not used.

But such improvement in cutting techniques is only possible through the increasingly erudite use of geometry. Rough casting only requires one view, the ground plane or the elevation, without ever having to manage both simultaneously. In our example, the ground track is sufficient in the first instance to start elevating the walls of the round tower up to where the vault begins; then, the arch elevation alone allows the voussoirs to be pre-cut in order to be fitted, the final shape being given later as shown above. Squaring on the other hand requires one to have the geometral projection of the dry-stone piece to be built, since corresponding ground plane and elevation portions must be drawn on the squared block of stone. Template cutting makes use of all the geometric operations associated with geometral representation, including the folding over of planes that occupy any position in space onto one of the reference planes.

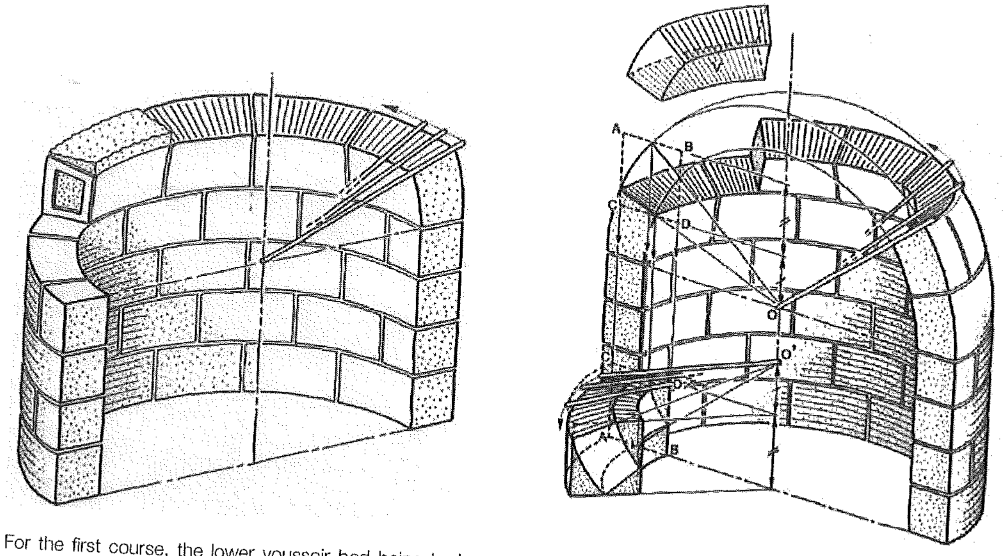
How does one go from one carving technique to another and what were the factors that impelled such evolution? According to a commonly accepted and defended hypothesis, notably by Viollet-le-Duc or Choisy, stone carving techniques probably originated in the East and were brought back to the West by crusaders. The development of stereotomy in the XIIth and XIIIth century, particularly in the South of France, is naturally the first argument in favour of this hypothesis. Indeed it is in Paleochristian Syria that erudite dry-stone architecture really began to flourish; the Romans, as did the Greeks, certainly built dry-stone arched and vaults, but no cupolas, and always avoided cradle-vault penetrations[8]. Erudite dry-stone architecture originated at the very boundaries of the Roman, and later Byzantine empires, in a zone where the most elaborate fortification systems were erected as protection from Persian invasions. The meeting in one and the same region, which had a long tradition of stone construction, of the knowledge of the best Roman architects and engineers, and specific needs in military architecture might explain how local masons perfected dry-stone construction techniques[9]. A few very beautiful hemispherical or quarter spherical perfectly built dry-stone vaults are a testimony to the fact that remarkable stereotomic know-how developed in this Middle-East region[10].

But did the crusaders find already evolved graphical techniques in the Near East, as suggested by Viollet-le-Duc ? Nothing is less certain. However remarkable Near East dry-stone architectural pieces might be, none absolutely requires the use of squaring or template tracing. The penetration of the cradle vaults in Pergamon can be achieved by

8 The only known exception, pointed out by Choisy, is a vaulted tomb in Pergamon (IInd century BC) comprising a cradle-vault penetration. Roman cupolas, such as the Pantheon cupola in Rome, do not have dry-stone joints but are mortared; it is the mortar that gives the whole construction its solidity upon drying, the exact shape of the stones or bricks used being of little importance.

9 See C. Mango, *Architecture byzantine*, Milan, Gallimard/Electa, 1993, pp. 55-66.

10 An example of this is the Qualblosch basilica (dating from the end of the 5th century), the apse of the oriental Qal'at Saman basilica (also dating from the 5th century), and the al-Mundhir audience hall in Resafa (6th century); cf. Mango, Op. Cit. Cupolas of the same type (very skilfully built with dry-stone joints) are also found in Armenian architecture of the 7th century (the T'alich church, the Mren cathedral and the Odzoun church). The Theodoric mausoleum (Ravenna, first half of the 6th century) is the only monument of the Italian peninsula that is comparable in terms of stereotomic virtuosity with the previously cited buildings, but it was almost certainly built by an architect originating from the Middle-East.



For the first course, the lower voussoir bed being horizontal and the voussoirs being in place as in rough casting, the upper bed can be cut using a rope or a pole that materialises the radius of the sphere (of the intrados and extrados) and the generator lines of the first cone. The rope allows drawing of the voussoir head templates as well as correct cutting of the dovela intrados spherical surface and the upper bed conical surface, as long as the outermost directing circle arc of this cone trunk has been drawn prior to this.

For the second course, the lower bed surface is cut first by materialising a cone that is equal but inverted with respect to the one that served to cut the upper face of the first-row voussoirs. As for the first row, the upper bed of the second-row voussoirs is cut after fitting the blocks of stone. The process merely needs to be reiterated for subsequent rows

Fig. 9: Principle of construction of a spherical cupola by the repointing technique («*taille à la perche*»).

rough casting and the spherical cupolas could have very well been carved by repointing technique («*à la perche*»), as illustrated in fig. 9.

The second argument that casts doubt on the idea that Crusaders found very elaborate graphical techniques in the Orient comes from examining the first known technical drawings pertaining to dry-stone architecture. The first explicit traces of construction drawings that have come to our knowledge, drawings engraved in stone, fail to show such geometrical virtuosity[11]. These drawings of corbels, windows and portals are generally no more than simple profiles. The huge drawing of the Western facade of the Reims cathedral, which superimposes ground track and elevation, is an exception. The history of graphical drawing seems to be first of all the history of practical techniques elaborated in order to avoid having to use drawing before being that of improvement on geometrical techniques.

11 The oldest drawings, dating from the end of the 12h century, are those of the Cistercian Abbey in Byland, Yorkshire. These drawings, which are true size drawings traced on large planar vertical or horizontal surfaces, were sometimes achieved, in England, in rooms known as «tracing houses» specifically built on the work site and generally destroyed after completion of the work; the one in York, for example, has been conserved. However, numerous drawings, dating from the 13th and 14th century, were drawn directly onto the walls or floor of buildings under construction, as in the Reims, Auxerre, Soissons, and Clermont-Ferrand cathedrals, for instance.

An Upheaval in Thinking about Construction

While it is possible to associate a period of reference with each stone cutting technique (Antiquity and the Roman period for rough casting, the Gothic period for squaring, the Renaissance for template cutting) nothing would be more erroneous than to imagine a sudden passage from one technique to the other. Even after the publication of stone cutting treatises[12], the three techniques remained in common use on worksites and it is the going back and forth from one to the other that led to the gradual development of geometrical drawings. In a sense, squaring and rough casting are reciprocal methods with respect to each other. Let us return to the example presented above and let us imagine the construction of the door in the round tower. Using rough casting, the tower is «drawn up» from its ground track, the voussoirs of the arch are fitted into place without taking into account the rotundity of the wall in which the door is located, and once the tower (or at least the level concerned) is finished, the voussoirs are re-cut. The chisel of the stonecutter then becomes a materialisation of the projection line onto the ground. Carrying out the same stone cutting work before the voussoir is fitted into place rather than after is tantamount to making an abstract rendering of this projection operation before construction. Geometral representation originated at the time when cutting by rough casting was giving way to squaring. As Choisy observed, *«in the Roman period, a*

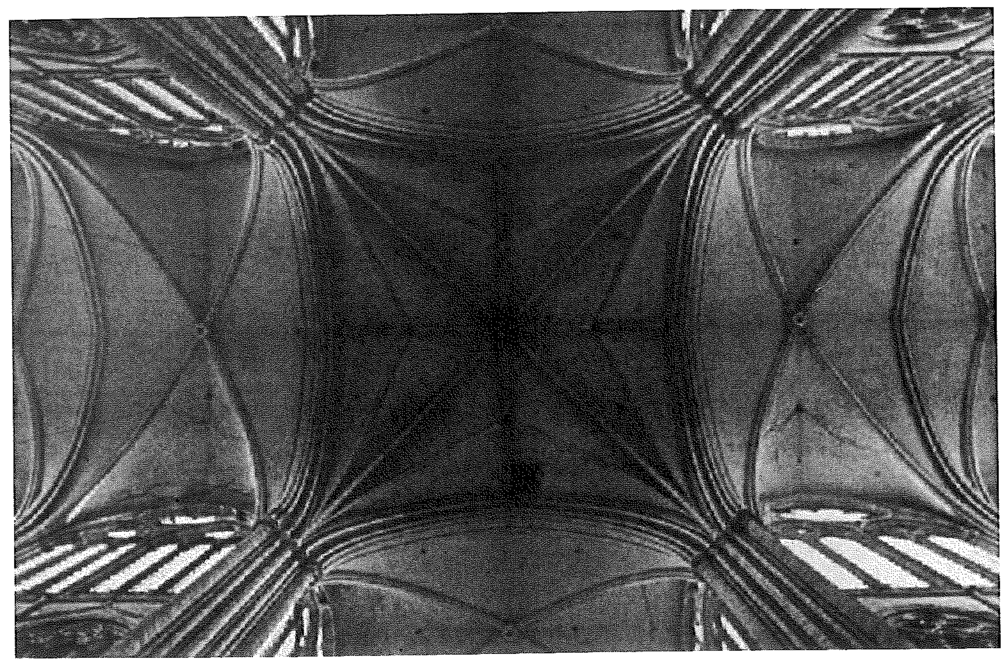


Fig. 10: Notre-Dame, Amiens Cathedral, 13th century, Robert de Luzarches arch.
For Gothic builders, it was the carried object that must impose the shape of the object that carries

12 The first stone cutting treatise was published by Philibert de L'Orme in 1567.

technique emerged, which would become an absolute rule in the Gothic era, and that is to fit the stone into place after it has been cut »[13]. Now, to «*fit the stone into place after it has been cut*» requires that geometral representation be adopted. It is undoubtedly no accident if the first geometral representation drawings and the term «foreman» appeared quasi simultaneously[14]. As for architecture, there is every reason to believe that for stereotomy, even «erudite» stereotomy, one began with building and drew afterwards. It is unlikely that improvement in geometrical drawings was what gave rise to perfecting of stone cutting techniques, but rather the reverse, at least in the beginning. Rough casting defines the projection, squaring starts from it, and the switching from one stone cutting technique to the other produces rather than derives from geometral representation.

Finally, the passage from rough casting to squaring was going to create a total upheaval in thinking about construction. Throughout Antiquity and the High Middle Ages, the construction of a building was thought in the direction in which it was being carried out, i.e. from the bottom up. For Gothic builders on the other hand, it was «*the carried object that (must) impose... the shape of the object that carries*», which requires that one think about the construction of a building from the top down. The point of an architectural construction is to cover an empty space, «*pillars and walls are merely, and should only be, means of obtaining an empty space*». Consequently, if the space to be built is a vaulted room, «*it is the vault that covers the empty space... that is the essential part of the structure; it is therefore the vault, its shape, extent and weight that govern the location, shape and resistance of support points. It logically follows that... the vault should be drawn first and that this drawing will govern the drawing of the pillars and walls*»[15]. This upheaval in thinking about construction is fundamental with respect to the problem of architectural representation because it introduces the operation of projection in a physical, material and concrete manner. While the plan is traced on the ground in Antiquity and Roman times, the building being rooted in the ground, the Gothic plan is seen from above and. takes on the meaning we give it today.

As it became the drawing «*according to the rules of the art*», the foreman's drawing par excellence, template cutting slowly began to override other techniques (at least in theory), relegating them to some sort of a-geometric prehistory. Nonetheless, rough casting and squaring have undoubtedly played a fundamental primordial role by inscribing the process of orthogonal projection into matter itself and making geometral projection concrete and tangible. Once this geometric operation was in place, it became possible to extend it, refine it and develop it, which is precisely what template cutting, carpentry or ironsmithery drawings would eventually do. It is only at the end of the XVIII[th] century that Gaspard Monge, geometer and founder of the Ecole Polytechnique, was to bring

13 A. Choisy, *Histoire de l'architecture*, Paris, 1899, t. II, p. 113.

14 Cf. J. -M. Pérouse de Montclos, *L'architecture à la Française, XVI, XVII, XVIII siècles*, Picard, Paris, 1982, p. 91 ; first mention of the *apparator* (foreman) dates back to 1292.

15 E. Viollet-le-Duc, *Dictionnaire raisonné de l'architecture française du XI au XVI siècle,* Paris, 1854-1868, article «Trait».

together these practical geometry drawings into a coherent and abstract doctrinal corpus that he was to call «descriptive geometry».

But the Monge's theory is not borne strait from those practical geometries. During the second half of the XVIII[th] century, will appeared what we can call a «scholastic stereotomy».

The Stereotomy as a scholastic discipline

Stereotomy constituted the expert construction technique par excellence, from the Middle Ages until the XVIII[th] century. Thus the education of engineers is going to include an apprenticeship in stereotomy. The richness of this construction technique, the superposition of the problems of a geometric, statics, esthetic, and economic order all met during the voussoir construction, are even going to make, at the time that engineering schools were set up in France, in the second half of the XVIII[th] century, one of the key disciplines in their curriculum. But depending on the objective of the training, the type of engineer desired, and the period, extremely dissimilar teachings were included under the same title. At the Ecole du Genie de Mezieres, one of the most prestigious European engineering schools of that time, teaching of stereotomy was not reduced to the strict utilitarian aspect of the construction technique. The essential objective of this course was the teaching of geometry and the visualization in space. The founder of the School, Nicolas de Chastillon, explicitly formulated this idea in an article of the regulations of 1754 :

> «*Independantly of the utility of the cutting of stone and wood presented by the different constructions of the King, these arts open such exact and precise knowledge on the drawing of the plans and profiles and on the manner of expressing the relief which must be represented, that one can consider them as Elements (of Euclid)*».

In the foreword of his *Treatise on Shadows in Geometric Drawings*, Chastillon repeats this idea :

> «*We have found nothing more proper for them (the Engineers) than to procure that perfect knowledge of design through the study of stone and wood cutting. Independant of the advantages which result from this study, relative to constructions of which the officers of engineering have the direction, one conceives easily that when one knows how to develop all the faces and knows all the angles of any stone used in a vault, a squinch, etc... or of a piece of timber used in the roof, a dome, a stairway, etc... one has easily the facility to develop a bastion, a semi-circle, a cavalier entrenchment, a battery, etc... that when one knows the representation of all these things in order to make them understood by others in the*

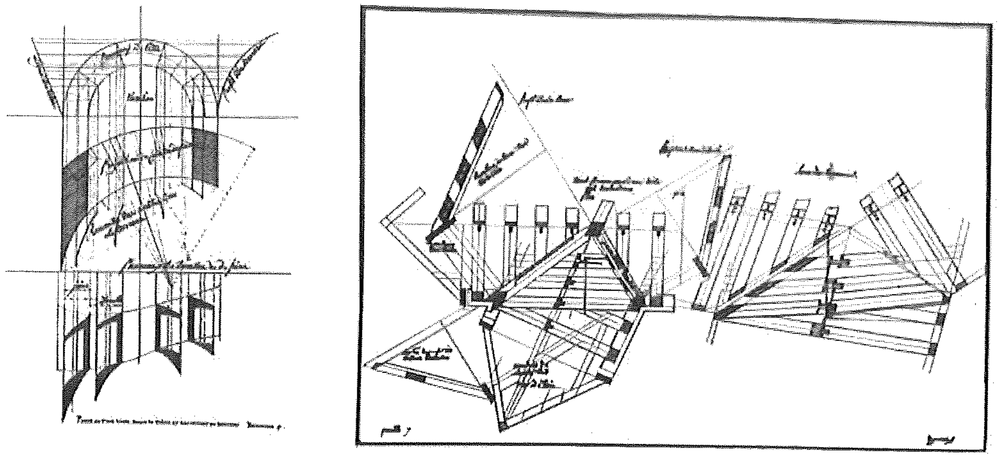


Fig. 11: The study of stone and wood cutting in the Ecole du Genie de Mezieres, Vigoureux's Drawings Portfolio, 1767

state of the representation as if they were already executed, and to combine the different structures in order to render them as perfect as they possibly can be»[16].

Here the founder of the Ecole du Genie de Mezieres expresses perfectly the pedagogic role confered on stereotomy in this school, the apprenticeship of space that its teaching permits, that sort of intellectual gymnastics susceptible to permit future engineers to mentally represent objects (eventually complex from a geometric point of view) not yet created. This teaching will be judged so fundamental that rather rapidly after the creation of the school, and in any case before the death of Chastillon in 1765, it will be placed in first year, before the course of construction. Here we observe a first sliding in the function of the teaching of stereotomy. If one admits that an engineer must receive a certain education concerning the apprenticeship of «visualization in space» then the choice of stereotomy as a vector is certainly judicious, and the arguments advanced by Chastillon are quite pertinent. The ease with which the stoneworkers or the stone cutters are capable of conceiving and tracing the most complex voussoirs is the best proof.

In 1768, when Monge took charge of the teaching of stone and wood cutting, as well as that of geometry, perspective or the drawing of shadows, he only systematized a principle of teaching put in place by his predecessors. Pushing the logic instituted by the founders of the Ecole du Genie de Mezieres to the limits, he looked to extract the geometric theory underlying stereotomy and introduce the rudiments of what he would call later descriptive geometry but which he still called «the method of cutting stones».

16. Archives de l'inspection du Génie, art. 18, sect. 1, §1, carton 1, pièce 9.

The teachings of Monge at the Ecole Polytechnique

At the time of the creation of the Ecole Polytechnique, in 1794, Monge had succeeded to some degree in the evolution of a process begun at the Ecole du Genie de Mezieres. Taking up again the idea according to which the training of engineers must include an apprenticeship in spatial representation of volumes, and surfaces, and their intersections, he confered this role, firstly to descriptive geometry and no longer uniquely to stereotomy. A scholastic discipline which was born in a school, by a school and for a school (but maybe one should say «in the Ecole Polytechnique, by the Ecole polytechnique, and for the Ecole polytechnique»), descriptive geometry allows the passage from one process of training by apprenticeship in little groups which was characteristic of the schools of the Ancien Regime, to an education in amphitheaters, with lectures, and practical exercises, which are no longer addressed to 20 students, but to 400 students. Descriptive geometry also stems from the «revolutionary method». A means to teach space in an accelerated way in relation to the former way of teaching stereotomy, an abstract language, minimal, rapid in the order of stenography, descriptive geometry permits a response to the urgent situation as for the education of an elite, which was the case of France at the moment of the creation of the Ecole Polytechnique.

Thus descriptive geometry occupies at the Ecole Polytechnique, the same place at stereotomy at the Ecole du Genie de Mezieres, and one can say at the same time that descriptive geometry is to the Ecole Polytechnique what stereotomy is to the Ecole du Genie de Mezieres but also that the Ecole Polytechnique is to the Ecole du Genie de Mezieres what descriptive geometry is to stereotomy. Monge never presents descriptive geometry as a new science of which he might be the founding father. Quite the contrary, he describes it as «*having been practiced for a great deal longer* (than Analysis) *and by many more people*». He even adds that descriptive geometry having been practiced «*by men whose time was precious, the* (graphical) *procedures were simplified and, instead of considering three planes, one got (thanks to projections) to only require two planes explicitly*»[17]. Thus, contrary to what is later going to be reproached Monge, the minimalist character of the diagram lines used in descriptive geometry is not the fruit of a mathematician's theoretical research but stems from the perfecting of practices over the years.

Although Monge does not cite any names, he is obviously referring to the drawings of stone carvers and carpenters. The privileged ties that descriptive geometry enjoys with various graphical techniques is made evident by the abundant examples Monge gives in the foundation course, which is constantly enriched by references to diverse techniques that are likely to use descriptive geometry.

17 G. Monge, *Géométrie descriptive, in Les Séances des écoles normales ...*, Paris, 1795, (reprinted in 1992, §5, p. 312).

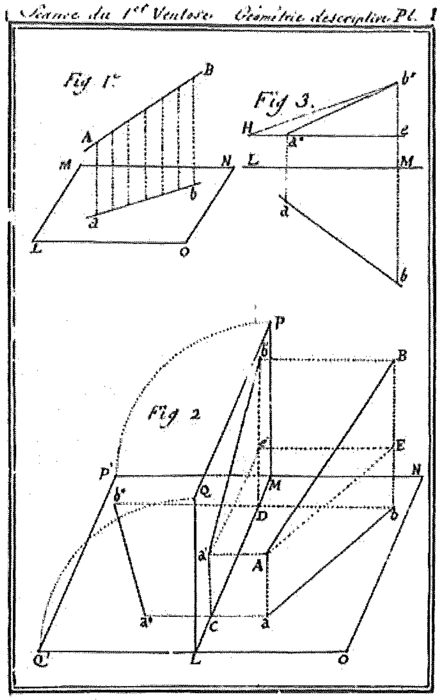


Fig. 12: Basic principle of space representation in descriptive geometry: representation of a point and of a line segment, Monge, *Géométrie descriptive*, Plate I.

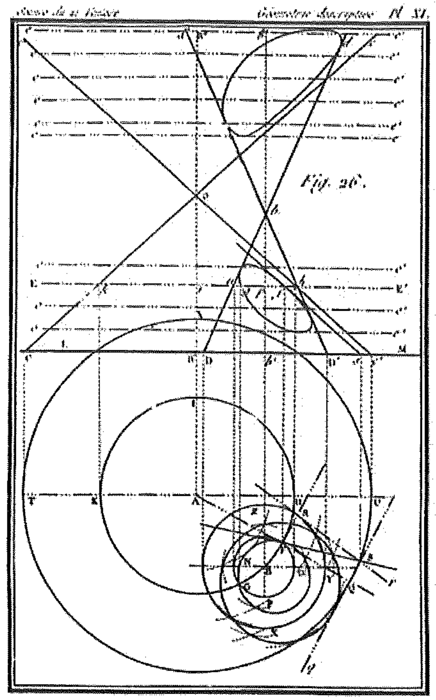


Fig. 13: Intersection of two cones; auxiliary plan method, Monge, *Géométrie descriptive*, Plate XI.

Descriptive geometry, a branch of scientific geometry

But Monge was not satisfied with extricating the essence of the stone workers' drawings. Descriptive geometry is both the conclusion of ancient methods of spatial representation and the point of departure for modern geometry. To show the distance and proximity between Monge's theory and the stone carvers techniques, I would like to focus on three points : the discovery of geometrical forms, the relationships between three-dimensional space and a two-dimensional plane, and the relationships between abstract and concrete.

Discovery of geometrical forms and operations

The diagram of descriptive geometry focuses not only on objects but also on the geometrical constructions used in the argument. More than the representation of space, the principal aim of descriptive geometry is the discovery of new volumes, of «*unknown forms, which necessarily result from the original forms given*»[18]. When we draw the diagram, we «construct» or determine the desired volume. Descriptive geometry makes it possible to advance step by step in this search with the help of a few simple principles, with two specific algorithms, according to Dupin, namely projection and rabatment.

18 J. N. P. Hachette, *Traité de géométrie descriptive...*, Paris, 1822, p. xj.

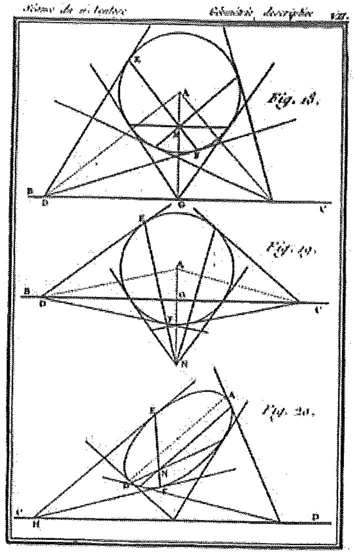
The strength of descriptive geometry in this process resides in its algorithmic and systematic resources allowing us to progress both surely and steadily.

It is not necessary to «see» the object in space before producing a diagram of it. Thanks to descriptive geometry we can discover a form that we were unable to imagine. Descriptive geometry allows us to represent three-dimensional objects on a sheet of paper, but for the representation of objects having simples forms it remains an extremely abstract method that is difficult to master compared with other ways of representing space, and axonometric projection in particular. Proof of this can be found in the fact that all the instructions for assembling furniture, games or other items delivered in kit form always use diagrams based on axonometric projection. Descriptive geometry only become irreplaceable when it is necessary to represent a complex object that cannot be imagined immediately such as, for example, the intersection of two given surfaces. Unless one has considerable experience (which has to be acquired one way or another), we do not know, in principle, the shape of the underside of a «twisted» staircase, the (spatial) nature of the intersection of two surfaces, the shape of a wedge-shaped stone at the intersection of two vaulted ceilings, the notches required to assemble several pieces of a framework, etc. The problem resides in representing these different objects on a sheet of paper without previously having a «mental representation» of them. It is naturally this stage that descriptive geometry makes it possible to complete, step by step, progressively, by a judicious use of their geometrical definition which contains the essential forming value of visualization in space. And it is this stage which disappears in CAD programs, which present the final result but conceal the intermediate processes. Descriptive geometry is a universal graphic method applicable to all particular techniques, because it theorizes two essential phases: the discovery of shapes and the representation of surfaces. Thus, it makes it possible to progress in the resolution of a practical problem (drawing of shadows, diagrams used in carpentry or stone-cutting, etc.) by a reciprocal passage between these two extremes.

But this two essential phases come strait from the stone carver techniques. By conceptualizing and theorizing the different stages leading to the discovery of a shape, descriptive geometry makes it possible to describe the geometrical operations required for the determination of that object. It «*reveals all its conceptions, all its operations and the graphic scales are a means for it to illustrate in space its approach and its results*»[19].

The algorithmic process use in descriptive geometry for the discovery of new volumes is very closed from the one used in carving by squaring, we just show supra. This proximity has two consequences: first, it proves that descriptive geometry is not only the theorisation of the geometrical methods used in the template method, but more generaly a theorisation of the carving methods ; second, it explain that descriptive

19. Ch. Dupin, *Développements de géométrie pour faire suite à la «Géométrie descriptive» et à la «Géométrie analytique» de M. Monge...*, Paris, p. 237.



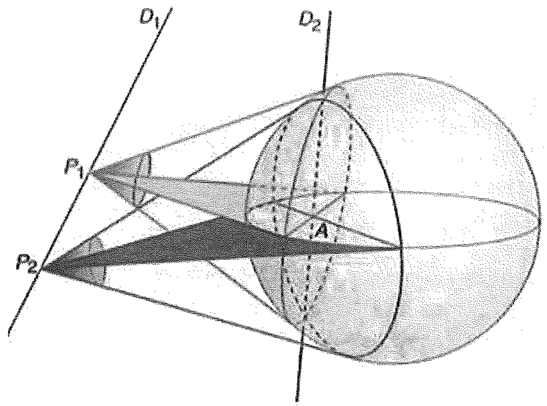


Fig. 14: A theorem of Ph. de La Hire : Monge, *Géométrie descriptive*, Plate VII.
The chord joining the points where tangents derived from a given point enter into contact with a circle pass through a fixed point when the point moves on a given straight line. Conversely, the tangents derived from the points of intersection of a straight line Δ and the circle cut one another at a point that moves along a straight line if Δ turns around a fixed point.

Let Π be the plane defined by the straight line Δ and the centre of the circle A. Monge considers the sphere centred at point A and having the same radius as the circle and the cones of revolution tangent to the sphere whose vertex moves along the straight line Δ. The cones and the sphere admit the same tangent plane P, containing the straight line Δ (Π is a plane of symmetry of the figure and for the rest of the argument we may only consider the volumes situated «above» Π). The point N where P comes into contact with the sphere belongs to all the circles of contact between the cones and the sphere ; these circles are always situated in the planes perpendicular to Π. If these volumes are projected onto Π, the circles of contact are projected on the chords of the circle which pass through N projection of N, which makes it possible to deduce the theorem.

geometry was able to take in charge the pedagogic role confered on stereotomy in the Ecole du Genie de Mezieres.

A diagram in descriptive geometry, by simultaneously representing the object and the geometrical constructions required to obtain the result, thus becomes an extremely valuable teaching aid. The person who draws the diagram can re-read what he has done, retrace his steps, correct if necessary an incorrect construction just as when we write out a demonstration or defend an idea we find it useful to re-read our earlier paragraphs. In a word, it enables us to learn, and not simply to repeat gestures. «*It is necessary to write down our discoveries as we make them* (to be able) *to retrace our steps with confidence... the new geometry can be used, simultaneously, to describe what the mind sees, and to write (and thereby to fix invariably) what the mind has seen*»[20]. This constitutes a fundamental difference with the graphic techniques that came before it, but also with modern computer tools for which the operations, based on matrix calculations, are carried out by the software and no longer form an integral part of the picture and are concealed behind the black box of the monitor.

20 Th. Olivier, *Traité de géométrie descriptive, théorique et appliquée*, Paris, 1843, p. VIII.

The reciprocal transition between three-dimensional space and a two-dimensional plane

Descriptive geometry is not only a geometrical tool used to project three-dimensional space onto a two-dimensional plane, as certain critics of descriptive geometry claim to think to the point of asserting, in a provocative statement, that descriptive geometry flattens space, and the brain along with it. The best definition of descriptive geometry is «the reciprocal transition between three-dimensional space and a two-dimensional plane». An example of this reciprocal transition is given by the demonstration given by Monge from a theorem of Ph. de La Hire (fig. 14).

In these demonstration, Monge uses descriptive geometry to achieve what Michel Chasles was to call «the intimate and systematic alliance between three-dimensional solids and two-dimensional figures»[21]. It is in this sense that descriptive geometry is the geometrical theory underlying architectural representation, irrespective of the tools used to obtain this representation, considering that the constant transfer between three-dimensional space and a two-dimensional plane are one of the characteristics of the architect's profession. It is also for this reason that architectural drawing is one of the principal sources of this geometrical theory.

The relationships between abstract/concrete or theory/practice

Descriptive geometry manages the relationships between abstract and concrete, or theory and practice, in an extremely original, and extremely powerful, manner. The very abstraction of this branch of geometry constitutes both its difficulty for students and its value as a training instrument. But it is by no means a purely abstract theory like other branches of mathematics owing to the diagrams drawn when practising it and the concrete applications it offers (if we adhere to the narrow definition usually given to it today) or that it contains, if we adopt Monge's definition, which is certainly more appropriate to its teaching in schools of architecture. The place granted to the applications of descriptive geometry in the curriculum of the first Ecole Polytechnique shows how Monge did not intend to limit engineers' training to the study of theoretical disciplines.

But the tracing of the diagrams themselves, unrelated to the applications, calls for a certain manual dexterity in the art of drawing: «*All the ostensible work will consist in the graphic constructions, in the drawings. These drawings, these constructions require considerable thought* (on the part of the students). *But there will be no moment purely devoted to this thinking. It will take place throughout the entire construction and the student, who will have exercised his intelligence and the dexterity of his hands, will have as the reward for this double work the exact description of the knowledge he will have acquired*»[22].

21 M. Chasles, *Aperçu historique sur l'origine et le développement des méthodes en géométrie...*, Bruxelles, 1837, p. 191.

22 G. Monge, «Procès verbaux des séances du conseil d'administration de l'Ecole centrale des travaux publics», *20. pluviose an III*, Archives de l'Ecole polytechnique.

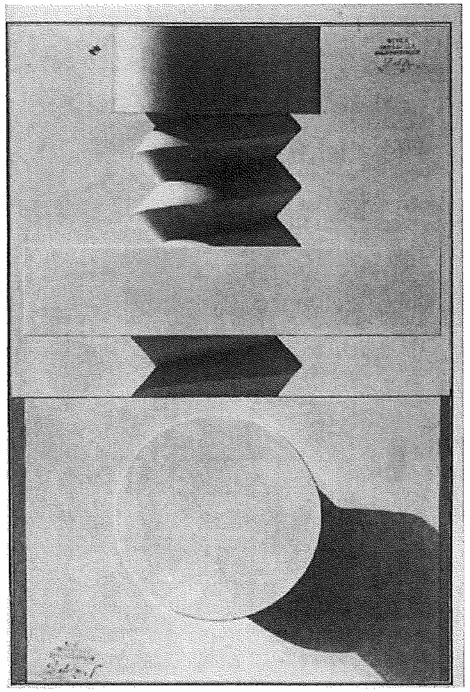


Fig. 15: Dalesme's Drawings Portfolio, 1812

In this way, descriptive geometry acquires the status of a discipline symbolizing the relationship between theoretical knowledge and practical know-how. Through the applications it makes possible, it remains a link, a meeting place, a confrontation between geometry and material reality in the form of stone, wood, concrete, metal or cloth, and this encounter is also one of the characteristics of the work of an architect.

There is no doubt that descriptive geometry is no longer, for mathematicians, the most promising branch of maths as far as new results are concerned. We can merely observe that after being practised for 150 years, it has been judged superfluous for an engineering syllabus. Has, then, the «language required for the man of genius who designs a project» become the «Latin of three-dimensional space», a dead language whose study is merely of cultural interest? Not necessarily. Because it lies at the heart of the transition between three- and two-dimensional space, at the heart of the relationship between drawing and mathematics and the tensions between geometry and the material world, descriptive geometry lies at the very heart of architecture and remains a key discipline in the training of future architects. It also remains a valuable tool for developing the ability of the student architects to imagine and invent three-dimensional space, and its study remains the most certain way to ensure that the architects of the future do not lose the mastery of their specific language

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