El presente volumen recoge artículos realizados a partir del material impartido en el curso de verano El Arte de la Piedra, teoría y práctica de la cantería de la II Summer School de la Universidad CEU San Pablo, celebrado en las instalaciones de la Escuela Politécnica Superior, entre el 9 y el 29 de julio de 2007, organizado por el Vicerrectorado de Relaciones Internacionales y dirigido por José Carlos Palacios Gonzalo y Alberto Sanjuán Álvarez.

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From the Stone Carver’s Techniques to Descriptive Geometry
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According to a clearly oversimplified though not totally unjustified view, the architect is first and foremost the person that can conceive and graphically translate a building project. The graphical techniques used in this process are so strongly associated with architectural function that in everyday language, the use of a triptych section-plane-elevation representation of a building tends to be referred to as an “architect’s drawing”. Such a representation is said to be “geometrical” according to the terminology specific to this art. Geometrical representation allows the various building trades in charge of erecting the building to understand and then proceed to build using all the necessary information in the graphical documents, including measurements of the various parts of the building. In order for the building trades to comply rigorously with the project imagined by the architect, both overall and in every detail, the graphical language used should be devoid of any ambiguity, which explains why representations with a conical perspective (analogous to a photograph) or an oblique parallel ( cavalier) projection (used for instance in instructions for the assembly of objects said as separate parts) are only used to supplement rather than substitute for geometrical representation.

The idea that the architect is a plan producer is relatively recent in the history of architecture. The first drawings clearly displaying articulation between two views are not to be found before the XVIth century and remain very exceptional in the XVIIth century. A drawing for the Milan cathedral, attributed to Antonio di Vincenzo and dated 1359, provides the first example where two distinct views, a ground plan and partial section,
are clearly articulated with each other1 (fig. 1). The majority of known architectural drawings from that time remain desperately flat and correspond to what we would call today a projection onto one plane. This is not to say that the designer did not conceive of the object in space but the latter being built by hand in manne concept, the crucial drawing step prior to construction was not indispensable given the workshops organization conditions, the constant presence of the architect, the degree of freedom left to masons during the work, etc. Principals lasting during Antiquity and the Roman period, there is nonetheless evidence for graphical production on the part of architects for over four thousand years2. But the tracing of a ground track, a vault profile, an elevation drawing (or even a ground plan and elevation) fail to constitute (so far) an geometrical representation. Indeed the latter not only requires that the three views be traced on the same scale, but also that they be thought of and articulated in relation to one another. Only then can one truly consider that a graphical spatial representation technique has been devised to meet the modern needs of architects and engineers. While the underlying concept in geometrical representation remains simple and only requires the use of relatively simple geometric tools (orthogonal projection essentially), this spatial representation mode is at the same time very abstract, which explains the additional use of more readily understandable representations. This also explains the rather mysterious and esoteric atmosphere surrounding the know-how of corporations using drawings based on axonomic projections.

While geometrical representation is nowadays associated with architectural drawing, throughout the Middle Ages and the Renaissance, it was first and foremost associated with the drawings of stone carvers. Hence the necessity for anyone wishing to address the study of proportions to have understood how to make measurements and thoroughly grasp how any object must be laid out in its ground plane and elevated, according to the method stone carvers use in their daily practice, writes Dürer3, for example. In order to show that geometrical representation indeed originates from stone carving techniques and understand how geometrical representation was literally "built" from such techniques, it is necessary to present them rapidly.

**Stone Carving Techniques**

There are three major stone carving techniques: rough casting, squaring and template cutting. They can be compared using the same dry-stone architectural structure4, namely a straight door in a round tower (fig. 3), which can be achieved by each of these methods.

Rough casting consists in casting the stones once they have been fitted in the vault or arch. The stones are roughly cut on the ground and only given their exact shape once they are in place. In the example considered, it is possible to carve the voussoirs without taking the tower into account, as the arch belonged to a flat wall, and to remove the extra stone afterwards (fig. 3).

Carving by squaring, also called very graphically «decoration», consists in «carving a stone... taking into account the height and depth that determine the limits of what needs to be removed, as if one stripped the imagined figure of its clothing»,5 (fig. 4). Starting from real size tracing of the ground track and elevation, the foreman6 traces the

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1. For a project to build the bell tower of the Fribourg-en-Brisgau cathedral at the beginning of the 14th century, the ground plan and elevation are drawn in direct correspondance, it is nonetheless worth noting that the first elements of double projection representation of corpore architecture details appear in the Viollet de Nonancour's sketch (middle of the 19th century). On this subject, see J. Sakovitch, "Espaces d'architecture, de la coupe des plans à la géométrie descriptive," Architectures, 1989, pp. 49-56.

2. The oldest testimony comes from a Mesopotamian statue, known as the Sardis statue of Gaia with architectural plan, where Gaia (governor of the city of Lagash) is holding on his lap a plan thought to be for the temple of Ninurta (Louvre Museum, circa 2000 BC). One of the most famous plans in the Middle of the 3rd century BC, which presents an extremely rare advantage for this particular period that it corresponds to a building that still exists.


4. In dry-stone architecture, vaults and arches are composed of arch stones or voussoirs with oblique sides via which they lean on adjoining voussoirs.


6. On a stone cutting workshop, the foreman is responsible for tracing scale drawings, i.e. drawings on a scale of 1 allowing...
Vousoir in the position it will occupy after assembly inside a rectangular parallelepiped with horizontal or vertical sides. Each vertex, or any given point, is then located by its projection onto two orthogonal faces of the envelope. In our example, in order to cut vousoir A (Fig. 5), the foreman traces out the portions of the ground track and elevation on the corresponding faces of the squared block of stone he started from. The vousoir is then cut using this tracing alone (Fig. 6). It is then a matter of cutting the stone so as to form a cylinder with vertical generator lines and director curves traced out on the horizontal planes', and similarly to form the cylinder with horizontal generator lines, thus allowing the desired vousoir to appear at the intersection of the two cylinders without the shape of each vousoir to be precisely determined.

7 The term ‘cylinder’, in accordance with its use in geometry, designates any surface defined by the set of generator lines parallel to a given direction and lying against a given curve (called the generator).

In the template method, the volume of each vousoir is determined from the surface of each of its sides. The vousoir is fitted in a minimal rectangular parallelepiped (Fig. 5, vousoir A), which now only has one plane direction parallel to one of the spatial references. In the example provided, the parallelepiped has two vertical faces and no horizontal faces. After having traced the face template as given by the elevation, the stonemason uses it to obtain the cylinder with horizontal generators. Any reference to the ground plane-elevation reference system having been lost at this point, it is necessary to have previously determined the exact geometrical shape of each of the vousoir sides in order to complete the work. More specifically, in the example shown in figure 7, one must having to predictetermine its shape. Naturally, at the time of cutting, one must avoid loosing the projection on one of the sides (the vertical one for instance) by cutting the entire vertical-generator cylinder in one go. In practice, the projections are traced on the two parallel faces and each half is cut one at a time, as indicated in figure 6.
But such improvement in cutting techniques is only possible through the increasingly erudite use of geometry. Rough casting only requires one view, the ground plane or the elevation, without ever having to manage both simultaneously. In our example, the ground track is sufficient in the first instance to start elevating the walls of the round tower up to where the vault begins; then, the arch elevation alone allows the voussoirs to be pre-cut in order to be fitted, the final shape being given later as shown above. Squaring on the other hand requires one to have the geometrical projection of the dry-stone piece to be built, since corresponding ground plane and elevation portions must be drawn on the squared block of stone. Template cutting makes use of all the geometric operations associated with geometric representation, including the folding over of planes that occupy any position in space onto one of the reference planes.

How does one go from one carving technique to another and what were the factors that impelled such evolution? According to a commonly accepted and defended hypothesis, notably by Viollet-le-Duc or Choisy, stone carving techniques probably originated in the East and were brought back to the West by crusaders. The development of stereotomy in the XIIth and XIIIth century, particularly in the South of France, is naturally the first argument in favour of this hypothesis. Indeed it is in Paleochristian Syria that erudite dry-stone architecture really began to flourish; the Romans, as did the Greeks, certainly built dry-stone arches and vaults, but no cupolas, and always avoided crenelated penetrations. Erudite dry-stone architecture originated at the very boundaries of the Roman, and later Byzantine, empires, in a zone where the most elaborate fortification systems were erected as protection from Persian invasions. The meeting in one and the same region, which had a long tradition of stone construction, of the knowledge of the best Roman architects and engineers, and specific needs in military architecture might explain how local masons perfected dry-stone construction techniques. A few very beautiful hemispherical or quarter spherical perfectly built dry-stone vaults are a testimony to the fact that remarkable stereotomic know-how developed in this Middle East region.

But did the crusaders find already evolved graphical techniques in the Near East, as suggested by Viollet-le-Duc? Nothing is less certain. However remarkable Near East dry-stone architectural pieces might be, none absolutely requires the use of squaring or template tracing. The penetration of the crenelated vaults in Pergamon can be achieved by

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8 The only known exception, pointed out by Choisy, is a vaulted tomb in Pasargades (7th century BC), containing a crenelated vault penetration; Roman cupolas, such as the Pantheon cupola in Rome, do not have dry-stone joints but are mortared. It is the masonry that gives the whole construction its solidity upon drying, the exact shape of the stones or bricks used being of little importance.

9 See C. Mengs, Architecture byzantine, Milan, Galland/Ecluse, 1955, pp. 35-60.

10 An example of this is the Quabaklich basilica (dating from the end of the 6th century), the apse of the original Great Samanid basilica (also dating from the 6th century), and the 6th-century Mausoleum in Recurse (7th century AD). Cupolas of the same type, very similar in truth with dry-stone joints, are also found in Armenian architecture of the 10th century (the Khachkars of the Tigranchach church, the Menagh chancel and the Ossian church). The Frankish mausoleum (Recurse, first half of the 6th century) is the only monument of the Frankish period that is comparable in terms of stereotomy to stereotomically dry-stone buildings, but it was almost certainly built under the inspiration of the West-East.
rough casting and the spherical cupola could have very well been carved by repointing technique (à la parche), as illustrated in fig. 9.

The second argument that casts doubt on the idea that Crusaders found very elaborate graphical techniques in the Orient comes from examining the first known technical drawings pertaining to dry-stone architecture. The first explicit traces of construction drawings that have come to our knowledge, drawings engraved in stone, fail to show such geometrical virtuosity. These drawings of corbels, windows and portals are generally no more than simple profiles. The huge drawing of the Western facade of the Reims cathedral, which superimposes ground track and elevation, is an exception. The history of graphical drawing seems to be first of all the history of practical techniques elaborated in order to avoid having to use drawing before being that of improvement on geometrical techniques.

11 These drawings, dating from the end of the 12th century, are those of the Certosa Abbey in Bologna, Yorkshire. These drawings, which are fine side drawings traced on large planar vertical or horizontal surfaces, were sometimes archived in the work; the arch in York, for example, is still preserved. However, numerous drawings, dating from the 13th and 14th centuries at Chorlton and the cathedral of Chartres, for instance.

12 The first stone cutting treatise was published by Philippe de L'Orme in 1567.
technique emerged, which would become an absolute rule in the Gothic era, and that is to fit the stone into place after it has been cut. Now, to fit the stone into place after it has been cut requires that geometrical representation be adopted. It is undoubtedly no accident if the first geometrical representation drawings and the term "foreman" appeared quite simultaneously. As for architecture, there is every reason to believe that for stereotomy, even "rudelle" stereotomy, one began with building and drew afterwards. It is unlikely that improvement in geometrical drawings was what gave rise to perfecting of stone cutting techniques, but rather the reverse, at least in the beginning. Rough casting defines the projection, squaring starts from it, and the switching from one stone cutting technique to the other produces rather than derives from geometrical representation.

Finally, the passage from rough casting to squaring was going to create a total upheaval in thinking about construction. Throughout Antiquity and the High Middle Ages, the construction of a building was thought in the direction in which it was being carried out, i.e., from the bottom up. For Gothic builders on the other hand, it was "the carried object that (must) impose... the shape of the object that centers", which requires that one think about the construction of a building from the top down. The point of an architectural construction is to cover an empty space, "pillars and walls are merely, and should only be, means of obtaining an empty space. Consequently, if the space to be built is a vaulted room,... it is the vault that covers the empty space... that is the essential part of the structure; it is therefore the vault, its shape, extent and weight that govern the location, shape and resistance of support points. It logically follows that... the vault should be drawn first and that this drawing will govern the drawing of the pillars and walls." This upheaval in thinking about construction is fundamental with respect to the problem of architectural representation because it introduces the operation of projection in a physical, material and concrete manner. While the plan is traced on the ground in Antiquity and Roman times, the building being rooted in the ground, the Gothic plan is seen from above and, taking on the meaning we give it today.

As it became the drawing "according to the rules of the art", the foreman's drawing par excellence, template cutting slowly began to override other techniques (at least in theory), relegating them to some sort of a-geometric prehistory. Nonetheless, rough casting and squaring have undoubtedly played a fundamental primordial role by inscribing the process of orthogonal projection into matter itself and making geometrical projection concrete and tangible. Once this geometrical operation was in place, it became possible to extend it, refine it and develop it, which is precisely what template cutting, carpentry or isometry drawings would eventually do. It is only at the end of the XVIIIth century that Gaspard Monge, geometrician and founder of the Ecole Polytechnique, was to bring together these practical geometry drawings into a coherent and abstract doctrinal corpus that he was to call "descriptive geometry."

But the Monge's theory is not born stray from those practical geometries. During the second half of the XVIIIth century, will appeared what we can call a "scholastic stereotomy."

The Stereotomy as a scholastic discipline

Stereotomy constituted the expert construction technique par excellence, from the Middle Ages until the XVIIIth century. Thus the education of engineers is going to include an apprenticeship in stereotomy. The richness of this construction technique, the superposition of the problems of a geometric, statics, esthetic, and economic order all met during the vousoir construction, are even going to make, at the time that engineering schools were set up in France, in the second half of the XVIIIth century, one of the key disciplines in their curriculum. But depending on the objective of the training, the type of engineer desired, and the period, extremely dissimilar teachings were included under the same title. At the Ecole du Genie de Mazarie, one of the most prestigious European engineering schools of that time, teaching of stereotomy was not reduced to the strict utilitarian aspect of the construction technique. The essential objective of this course was the teaching of geometry and the visualization in space. The founder of the School, Nicolas de Chastillon, explicitly formulated this idea in an article of the regulations of 1754:

"Independently of the utility of the cutting of stone and wood presented by the different constructions of the King, these arts open such exact and precise knowledge on the drawing of the plans and profiles and on the manner of expressing the relief which must be represented, that one can consider them as Elements of (Enseignement)."

In the foreword of his Treatise on Shadows in Geometric Drawings, Chastillon repeats this idea:

"We have found nothing more proper for them (the Engineers) than to procure that perfect knowledge of design through the study of stone and wood cutting. Independent of the advantages which result from this study, relative to constructions of which the officers of engineering have the direction, one conceives easily that when one knows how to develop all the faces and knows all the angles of any stone used in a vault, a squinch, etc... or of a piece of timber used in the roof, a dome, a stairway, etc... one has easily the facility to develop a bastion, a semi-circle, a cavalier entrenchment, a battery, etc... that when one knows the representation of all these things in order to make them understood by others in the"
The teachings of Monge at the École Polytechnique

At the time of the creation of the École Polytechnique, in 1794, Monge had succeeded to some degree in the evolution of a process begun at the Ecole du Génie de Meudres. Taking up again the idea according to which the training of engineers must include an apprenticeship in spatial representation of volumes, and surfaces, and their intersections, he conferred this role, firstly to descriptive geometry and no longer uniquely to stereotomy. A scholastic discipline which was born in a school, by a school and for a school (but maybe one should say «in the École Polytechnique, by the École Polytechnique, and for the École Polytechnique»), descriptive geometry allows the passage from one process of training by apprenticeship in little groups which was characteristic of the schools of the Ancien Régime, to an education in amphitheatres, with lectures, and practical exercises, which are no longer addressed to 20 students, but to 400 students. Descriptive geometry also stems from the «revolutionary method»: A means to teach space in an accelerated way in relation to the former way of teaching stereotomy, an abstract language, minimal, rapid in the order of stenography, descriptive geometry permits a response to the urgent situation as for the education of an elite, which was the case of France at the moment of the creation of the École Polytechnique.

Thus descriptive geometry occupies at the École Polytechnique, the same place at stereotomy at the Ecole du Génie de Meudres, and one can say at the same time that descriptive geometry is to the École Polytechnique what stereotomy is to the Ecole du Génie de Meudres but also that the École Polytechnique is to the Ecole du Génie de Meudres what descriptive geometry is to stereotomy. Monge never presents descriptive geometry as a new science of which he might be the founding father. Quite the contrary, he describes it as «having been practiced for a great deal longer (than Analysis)» and by «many more people». He even adds that descriptive geometry having been practiced «by men whose time was precious, the (graphical) procedures were simplified and, instead of considering three planes, one got (thanks to projections) to only require two planes explicitly»17. Thus, contrary to what is later going to be reproached Monge, the minimalist character of the diagram lines used in descriptive geometry is not the fruit of a mathematician's theoretical research but stems from the perfecting of practices over the years.

Although Monge does not cite any names, he is obviously referring to the drawings of stone carvers and carpenters. The privileged ties that descriptive geometry enjoys with various graphical techniques is made evident by the abundant examples Monge gives in the foundation course, which is constantly enriched by references to diverse techniques that are likely to use descriptive geometry.


The strength of descriptive geometry in this process resides in its algorithmic and systematic resources allowing us to progress both surely and steadily.

It is not necessary to «see» the object in space before producing a diagram of it. Thanks to descriptive geometry we can discover a form that we were unable to imagine. Descriptive geometry allows us to represent three-dimensional objects on a sheet of paper, but for the representation of objects having simple forms it remains an extremely abstract method that is difficult to master compared with other ways of representing space, and axonometric projection in particular. Proof of this can be found in the fact that all the instructions for assembling furniture, games or other items delivered in kit form always use diagrams based on axonometric projection. Descriptive geometry only become indispensable when it is necessary to represent a complex object that cannot be imagined immediately such as, for example, the intersection of two given surfaces. Unless one has considerable experience (which has to be acquired one way or another), we do not know, in principle, the shape of the underside of a «twisted» staircase, the (spatial) nature of the intersection of two surfaces, the shape of a wedge-shaped steeple at the intersection of two vaulted ceilings, the notches required to assemble several pieces of a framework, etc. The problem resides in representing these different objects on a sheet of paper without previously having a «mental representation» of them. It is naturally this stage that descriptive geometry makes it possible to complete, step by step, progressively, by a judicious use of their geometrical definition which contains the essential forming value of visualization in space. And it is this stage which disappears in CAD programs, which present the final result but conceal the intermediate processes.

Descriptive geometry is a universal graphic method applicable to all particular techniques, because it theorizes two essential phases: the discovery of shapes and the representation of surfaces. Thus, it makes it possible to progress in the resolution of a practical problem (drawing of shadows, diagrams used in carpentry or stone-cutting, etc.) by a reciprocal passage between these two extremes.

But this two essential phases come straight from the stone carver's techniques. By conceptualizing and theorizing the different stages leading to the discovery of a shape, descriptive geometry makes it possible to describe the geometrical operations required for the determination of that object. It «reveals all its conceptions, all its operations and the graphic scales as a means for it to illustrate in space its approach and its results».

The algorithmic process used in descriptive geometry for the discovery of new volumes is very closed from the one used in carving by squaring, we just show supra. This proximity has two consequences: first, it proves that descriptive geometry is not only the theorisation of the geometrical methods used in the template methods, but more generally a theorisation of the carving methods; second, it explain that descriptive...
The reciprocal transition between three-dimensional space and a two-dimensional plane

Descriptive geometry is not only a geometrical tool used to project three-dimensional space onto a two-dimensional plane, as certain critics of descriptive geometry claim to think to the point of asserting, in a provocative statement, that descriptive geometry flattens space, and the brain along with it. The best definition of descriptive geometry is "the reciprocal transition between three-dimensional space and a two-dimensional plane." An example of this reciprocal transition is given by Monge from a theorem of Ph. de La Hire (fig. 14).

In this demonstration, Monge uses descriptive geometry to achieve what Michel Charles was to call "the intimate and systematic alliance between three-dimensional solids and two-dimensional figures." It is in this sense that descriptive geometry is the geometrical theory underlying architectural representation, irrespective of the tools used to obtain this representation, considering that the constant transfer between three-dimensional space and a two-dimensional plane are one of the characteristics of the architect's profession. It is also for this reason that architectural drawing is one of the principal sources of this geometrical theory.

The relationships between abstract/concrete or theory/practice

Descriptive geometry manages the relationships between abstract and concrete, or theory and practice, in an extremely original, and extremely powerful, manner. The very abstraction of this branch of geometry constitutes both its difficulty for students and its value as a training instrument. But it is by no means a purely abstract theory like other branches of mathematics owing to the diagrams drawn when practising it and the concrete applications it offers (if we adhere to the narrow definition usually given to it today) or that it contains, if we adopt Monge's definition, which is certainly more appropriate to its teaching in schools of architecture. The place granted to the applications of descriptive geometry in the curriculum of the first Ecole Polytechnique shows how Monge did not intend to limit engineers' training to the study of theoretical disciplines.

But the tracing of the diagrams themselves, unrelated to the applications, calls for a certain manual dexterity in the art of drawing: "All the considerable work will consist in the graphic constructions, in the drawings, these constructions require considerable thought (on the part of the students). But there will be no moment purely devoted to this thinking: it will take place throughout the entire construction and this student, who will have exercised his intelligence and the dexterity of his hands, will have as the reward for this double work the exact description of the knowledge he will have acquired."

In this way, descriptive geometry acquires the status of a discipline symbolizing the relationship between theoretical knowledge and practical know-how. Through the applications it makes possible, it remains a link, a meeting place, a confrontation between geometry and material reality in the form of stone, wood, concrete, metal or cloth, and this encounter is also one of the characteristics of the work of an architect.

There is no doubt that descriptive geometry is no longer, for mathematicians, the most promising branch of maths as far as new results are concerned. We can merely observe that after being practised for 150 years, it has been judged superfluous for an engineering syllabus. Has, then, the «language required for the main of genius who designs a project» become the «Latin of three-dimensional space», a dead language whose study is merely of cultural interest? Not necessarily. Because it lies at the heart of the transition between three- and two-dimensional space, at the heart of the relationship between drawing and mathematics and the tensions between geometry and the material world, descriptive geometry lies at the very heart of architecture and remains a key discipline in the training of future architects. It also remains a valuable tool for developing the ability of the student architect to imagine and invent three-dimensional space, and its study remains the most certain way to ensure that the architects of the future do not lose the mastery of their specific language.

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